On logarithmic Sobolev inequalities for normal martingales

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Abstract.
Let $(Z_t)_{t\in\mathbb{R}_+}$ be a martingale in L^4 having the chaos representation property and angle bracket $d\langle Z_t, Z_t\rangle = dt$. We show that the positive functionals F of $(Z_t)_{t\in\mathbb{R}_+}$ satisfy the modified logarithmic Sobolev inequality

$$E[F\log F] - E[F]\log E[F] \leq \frac{1}{2}E\left[\frac{1}{F}\int_0^\infty (2-i_t)(D_tF)^2dt\right],$$

where D is the gradient operator defined by lowering the degree of multiple stochastic integrals with respect to $(Z_t)_{t\in\mathbb{R}_+}$ and $(i_t)_{t\in\mathbb{R}_+} \subset \{0,1\}$ is a process given by the structure equation satisfied by $(Z_t)_{t\in\mathbb{R}_+}$.

Résumé.
Soit $(Z_t)_{t\in\mathbb{R}_+}$ une martingale dans L^4 qui satisfait la propriété de représentation chaotique, avec $d\langle Z_t, Z_t\rangle = dt$. On montre que les fonctionnelles positives F de $(Z_t)_{t\in\mathbb{R}_+}$ satisfont l'inégalité de Sobolev logarithmique modifiée

$$E[F\log F] - E[F]\log E[F] \leq \frac{1}{2}E\left[\frac{1}{F}\int_0^\infty (2-i_t)(D_tF)^2dt\right],$$

où D est l'opérateur gradient qui abaisse le degré des intégrales stochastiques multiples par rapport à $(Z_t)_{t\in\mathbb{R}_+}$, et $(i_t)_{t\in\mathbb{R}_+} \subset \{0,1\}$ est un processus donné par l'équation de structure satisfaite par $(Z_t)_{t\in\mathbb{R}_+}$.

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1 Introduction

The multiple stochastic integrals with respect to martingales having deterministic angle bracket dt (i.e. *normal* martingales) share the same orthogonality and norm properties. As a consequence, a number of common properties hold for all such martingales, and in particular for Brownian motion, the compensated Poisson process and Azéma's martingales. Examples of such properties are the coincidence of the divergence operator with the stochastic integral on adapted processes (3), the Clark formula (4), and variance and spectral gap inequalities (5). Although the second moments of such martingales are the same, higher order moments may differ. In fact the structure of each martingale implies a particular multiplication formula for multiple stochastic integrals, see § IV.3 of [10] and [12], which corresponds to a particular probabilistic interpretation of Fock space. In practice, few properties of chaos expansions remain common to all such martingales, for example the gradient operator D defined by lowering the degree of multiple stochastic integrals satisfies the chain rule of derivation only in the Brownian case.

The entropy of a random variable F under a given probability measure π, defined as

$$\mathrm{Ent}_\pi[F] = E_\pi[F \log F] - E_\pi[F] \log E_\pi[F],$$

is independent of the dimension of the probability space. The variance and entropy operators share the same product property, cf. Prop. 2.2 of [8]. This makes the entropy a good candidate in order to states inequalities that are independent of the probabilistic interpretation chosen for the Fock space.

Corollary 5.3 of [8] (see also [3]) states that

$$\mathrm{Ent}_\pi[f(Y)] \leq \theta E_\pi \left[\frac{1}{f(Y)} (f(Y+1) - f(Y))^2 \right], \tag{1}$$

where Y is a Poisson distributed random variable on $\mathbb{N}$ with mean $\theta > 0$, and it is pointed out in [8] that the constant θ is the best possible. This inequality has been extended in [1], [2], [13], [14], to functionals of the Poisson process. Although the proof of (1) relies on the particularities of the Poisson law, its extension will appear to be valid not only on Poisson space but also for a large family of normal martingales, and distributions: the law of e.g. the Azéma martingale is connected to the arcsine law, cf. [6] and Ch. 15 of [15].

In Sect. 2 we will show that the proof of modified logarithmic Sobolev inequalities on Poisson space of [1], [2], [3], [4] extends to the general setting of normal martingales, see Cor. 1. We also consider the extension, in the context of normal martingales, of the inequalities given in [14], cf. Prop. 1. The case of normal martingales satisfying deterministic structure equations is given particular attention in Sect. 3.

2 Modified logarithmic Sobolev inequality for normal martingales

Let $(Z_t)_{t\in\mathbb{R}_+}$ be a martingale such that

(i) $(Z_t)_{t\in\mathbb{R}_+}$ has deterministic angle bracket $d\langle Z_t, Z_t\rangle = dt$.

We denote by $(\mathcal{F}_t)_{t\in\mathbb{R}_+}$ the filtration generated by $(Z_t)_{t\in\mathbb{R}_+}$. The multiple stochastic integral $I_n(f_n)$ is defined as

$$I_n(f_n) = n!\int_0^\infty \int_0^{t_n^-} \cdots \int_0^{t_2^-} f_n(t_1,\ldots,t_n)dZ_{t_1}\cdots dZ_{t_n}, \quad f_n \in L^2(\mathbb{R}_+)^{\circ n}, \;\; n \geq 1,$$

with

$$E_\pi[I_n(f_n)I_m(g_m)] = n! 1_{\{n=m\}}\langle f_n, g_m\rangle_{L^2(\mathbb{R}_+)^{\circ n}}. \tag{2}$$

We assume that

(ii) $(Z_t)_{t\in\mathbb{R}_+}$ has the chaos representation property,

i.e. every $F \in L^2(\Omega, \mathcal{F}, \pi)$ has a decomposition as $F = \sum_{n=0}^\infty I_n(f_n)$. A martingale satisfying (i) is called a normal martingale in [5]. Let $D : \mathrm{Dom}(D) \longrightarrow L^2(\Omega \times \mathbb{R}_+, d\pi \times dt)$ denote the closable, unbounded gradient operator defined as

$$D_t F = \sum_{n=1}^\infty n I_{n-1}(f_n(*,t)), \quad d\pi \times dt - a.e.,$$

with $F = \sum_{n=0}^\infty I_n(f_n)$. The adjoint δ of D is defined by the duality

$$E_\pi[F\delta(u)] = E_\pi[\langle DF, u\rangle_{L^2(\mathbb{R}_+)}], \quad F \in \mathrm{Dom}(D), \; u \in \mathrm{Dom}(\delta),$$

and it coincides with the stochastic integral with respect to $(Z_t)_{t\in\mathbb{R}_+}$ for every predictable square-integrable process $(u(t))_{t\in\mathbb{R}_+}$, cf. Prop. 4.4 of [9]:

$$\delta(u) = \int_0^\infty u(t)dZ_t. \tag{3}$$

The Clark formula is a consequence of the chaos representation property for $(Z_t)_{t\in\mathbb{R}_+}$, see e.g. [9], and states that any $F \in \mathrm{Dom}(D) \subset L^2(\Omega, \mathcal{F}, P)$ has a representation

$$F = E_\pi[F] + \int_0^\infty E_\pi[D_t F \mid \mathcal{F}_{t-}] dZ_t. \tag{4}$$

It admits a simple proof via the chaos expansion of F:

$$\begin{aligned} F &= E_\pi[F] + \sum_{n=1}^\infty n! \int_0^\infty \int_0^{t_n^-} \cdots \int_0^{t_2^-} f_n(t_1, \ldots, t_n) dZ_{t_1} \cdots dZ_{t_n} \\ &= E_\pi[F] + \sum_{n=1}^\infty n \int_0^\infty I_{n-1}(f_n(*, t_n) 1_{\{* < t_n\}}) dZ_{t_n} = E_\pi[F] + \int_0^\infty E_\pi[D_t F \mid \mathcal{F}_{t-}] dZ_t. \end{aligned}$$

The Clark formula shows the spectral gap inequality

$$\mathrm{var}_\pi(F) \leq E_\pi[\|DF\|^2_{L^2(\mathbb{R}_+)}]. \tag{5}$$

The spectral decomposition of δD is completely known in terms of multiple stochastic integrals since $\delta D I_n(f_n) = n I_n(f_n)$, $f_n \in L^2(\mathbb{R}_+)^{\circ n}$. However, apart from the Brownian and Poisson cases, such integrals may not be expressed as polynomials, see [12]. If $(Z_t)_{t\in\mathbb{R}_+}$ is in L^4 then the chaos representation property implies that it satisfies the structure equation

$$d[Z_t, Z_t] = dt + \phi_{t^-} dZ_t, \quad t \in \mathbb{R}_+, \tag{6}$$

where $(\phi_t)_{t\in\mathbb{R}_+}$ is a predictable square-integrable process. Let $i_t = 1_{\{\phi_t = 0\}}$ and $j_t = 1 - i_t = 1_{\{\phi_t \neq 0\}}$, $t \in \mathbb{R}_+$. The continuous part of $(Z_t)_{t\in\mathbb{R}_+}$ is given by $dZ_t^c = i_t dZ_t$ and the eventual jump of $(Z_t)_{t\in\mathbb{R}_+}$ at time $t \in \mathbb{R}_+$ is given as $\Delta Z_t = \phi_t$ on $\{\Delta Z_t \neq 0\}$, $t \in \mathbb{R}_+$, see [6], p. 70.

In the following two cases, we have the chaotic representation property for $(Z_t)_{t\in\mathbb{R}_+}$ satisfying (6):

a) $(\phi_t)_{t\in\mathbb{R}_+}$ is deterministic. Then from Prop. 4 of [6], $(Z_t)_{t\in\mathbb{R}_+}$ can be represented as

$$dZ_t = i_t dB_t + \phi_t (dN_t - \lambda_t dt), \quad t \in \mathbb{R}_+, \quad Z_0 = 0, \tag{7}$$

with $\lambda_t = (1 - i_t) 1/\phi_t^2$, $t \in \mathbb{R}_+$, where $(B_t)_{t\in\mathbb{R}_+}$ is a standard Brownian motion, and $(N_t)_{t\in\mathbb{R}_+}$ a Poisson process independent of $(B_t)_{t\in\mathbb{R}_+}$, with intensity $\nu_t = \int_0^t \lambda_s ds$, $t \in \mathbb{R}_+$, cf. Prop. 4 of [6].

b) Azéma martingales where $\phi_t = \beta Z_t$, $\beta \in [-2, 0[$, see Prop. 6 of [6].

We now show that the modified logarithmic Sobolev inequality stated in [1] for the Poisson process extends to all normal martingales in L^4 with the chaos representation property, that is in particular to the Azéma martingale. We proceed by first stating the analog of the logarithmic Sobolev of [13], [14]. Let

$$\Psi(u, v) = (u + v)\log(u + v) - u \log u - (1 + \log u)v, \quad u, u + v > 0.$$

Proposition 1 *Let $F \in \mathrm{Dom}(D)$ be bounded and $\mathcal{F}_T$-measurable, with $F > \eta$ for some $\eta > 0$. We have*

$$\mathrm{Ent}_\pi[F] \leq E_\pi\left[\int_0^T j_t \frac{1}{\phi_t^2}\Psi(F, \phi_t D_t F)dt + \frac{1}{2}\int_0^T i_t \frac{1}{F}(D_t F)^2 dt\right]. \tag{8}$$

Proof. We follow [2] and [14]. Let $M_t = E_\pi[F \mid \mathcal{F}_t]$, $0 \leq t \leq T$. We have the predictable representation

$$M_T = M_0 + \int_0^T H_t dZ_t,$$

with $H_t = E_\pi[D_t F \mid \mathcal{F}_{t^-}]$, $0 \leq t \leq T$. The Itô formula for $(Z_t)_{t\in\mathbb{R}_+}$, see Prop. 2 of [6] states that for $f \in \mathcal{C}^2(\mathbb{R})$,

$$\begin{aligned} f(M_T) - f(M_0) &= \int_0^T \frac{f(M_{t^-} + \phi_t H_t) - f(M_{t^-})}{\phi_t} dZ_t \\ &\quad + \int_0^T \frac{f(M_{t^-} + \phi_t H_t) - f(M_{t^-}) - \phi_t H_t f'(M_{t^-})}{\phi_t^2} dt. \end{aligned}$$

If $\phi_t = 0$ the terms $(f(M_{t^-} + \phi_t H_t) - f(M_{t^-}))/\phi_t$ and $(f(M_{t^-} + \phi_t H_t) - f(M_{t^-}) - \phi_t H_t f'(M_{t^-}))/\phi_t^2$ have to be replaced by their limits as $\phi_t \to 0$, that is $H_t f'(M_{t^-})$ and $\frac{1}{2}H_t^2 f''(M_{t^-})$ respectively. Since $(M_t)_{t\in\mathbb{R}_+}$ is uniformly bounded from below by a strictly positive constant, we may apply this formula to $f(x) = x \log x$ to obtain:

$$\begin{aligned} &F \log F - E_\pi[F] \log E_\pi[F] \\ &= \int_0^T j_t \frac{(M_{t^-} + \phi_t H_t)\log(M_{t^-} + \phi_t H_t) - M_{t^-}\log M_{t^-}}{\phi_t} dM_t + \int_0^T i_t H_t f'(M_t) dM_t \\ &\quad + \int_0^T j_t \frac{(M_{t^-} + \phi_t H_t)\log(M_{t^-} + \phi_t H_t) - M_{t^-}\log M_{t^-} - \phi_t H_t(1 + \log M_{t^-})}{\phi_t^2} dt \\ &\quad + \frac{1}{2}\int_0^T i_t \frac{H_t^2}{M_t} dt, \end{aligned}$$

with $M_t = M_{t^-} + \phi_t H_t > \eta$, $0 \le t \le T$. We have

$$\begin{aligned}
\mathrm{Ent}_\pi[F] &= E_\pi\left[\int_0^T j_t \frac{1}{\phi_t^2}\Psi(M_{t^-}, \phi_t H_t)dt\right] + \frac{1}{2}E\left[\int_0^T i_t \frac{H_t^2}{M_t}dt\right] \\
&\le E_\pi\left[\int_0^T j_t E_\pi\left[\frac{1}{\phi_t^2}\Psi(F, \phi_t D_t F) \mid \mathcal{F}_t\right]dt\right] + \frac{1}{2F}E\left[\int_0^T i_t (D_t F)^2 dt\right] \\
&= E_\pi\left[\int_0^T j_t \frac{1}{\phi_t^2}\Psi(F, \phi_t D_t F)dt + \frac{1}{2F}\int_0^T i_t (D_t F)^2 dt\right],
\end{aligned}$$

where we applied Jensen's inequality:

$$\Psi(M_{t^-}, \phi_t H_t) \le E_\pi\left[\Psi(F, \phi_t D_t F) \mid \mathcal{F}_t\right]$$

to the convex function Ψ as in [13], and the Cauchy-Schwarz inequality

$$(E_\pi[i_t D_t F \mid \mathcal{F}_t])^2 \le E_\pi\left[\frac{1}{F} i_t (D_t F)^2 \mid \mathcal{F}_t\right] E_\pi[F \mid \mathcal{F}_t],$$

to $i_t D_t F$. $\square$

The modified logarithmic Sobolev inequality is obtained as a Corollary of Prop. 1.

Corollary 1 *Let $F \in \mathrm{Dom}(D)$ be bounded and $\mathcal{F}_T$-measurable, with $F > \eta$ for some $\eta > 0$. We have*

$$\mathrm{Ent}_\pi[F] \le \frac{1}{2}E_\pi\left[\frac{1}{F}\int_0^T (2 - i_t)(D_t F)^2 dt\right]. \tag{9}$$

Proof. We apply Prop. 1 with the inequality $\Psi(u, v) \le |v|^2/u$, $u > 0$, $u + v > 0$, cf. [2] and Cor. 2.1 of [14]:

$$\begin{aligned}
\mathrm{Ent}_\pi[F] &\le E_\pi\left[\int_0^T j_t \frac{1}{\phi_t^2}\Psi(F, \phi_t D_t F)dt + \frac{1}{2F}\int_0^T i_t (D_t F)^2 dt\right] \\
&\le \frac{1}{2}E_\pi\left[\frac{1}{F}\int_0^T (2 - i_t)(D_t F)^2 dt\right].
\end{aligned}$$

$\square$

Another proof of (9) consists in using the bound $b \log b - a \log a - (b - a)(1 + \log a) \le (b - a)^2/a$, $a, b > 0$ directly as in [2], Th. 4.1.

Corollary 2 *Let $F \in \mathrm{Dom}(D)$ be bounded and $\mathcal{F}_T$-measurable, with $F > \eta$ for some $\eta > 0$. We have*

$$\mathrm{Ent}_\pi[F] \le E_\pi\left[\int_0^T j_t \frac{D_t F}{\phi_t}(\log(F + \phi_t D_t F) - \log F)dt + \frac{1}{2F}\int_0^T i_t (D_t F)^2 dt\right]. \tag{10}$$

Proof. We apply Prop. 1 and the bound $\Psi(u,v) \leq v(\log(u+v) - \log u)$, $u > 0$, $u + v > 0$, as in Cor. 2.2 of [14]. $\square$

For the Azéma martingale with parameter $\beta \in [-2, 0[$ we have $i_t = 0$ a.e., hence

$$\mathrm{Ent}_\pi[F] \leq E_\pi\left[\int_0^T \frac{1}{\beta^2 Z_t^2}\Psi(F, \beta Z_t D_t F)dt\right] \leq E_\pi\left[\int_0^T \frac{1}{F}(D_t F)^2 dt\right],$$

and from Cor. 2:

$$\mathrm{Ent}_\pi[F] \leq E_\pi\left[\int_0^T \frac{D_t F}{\beta Z_t}(\log(F + \beta Z_t D_t F) - \log F)dt\right].$$

3 Deterministic structure equations

In this section, $(\phi_t)_{t\in\mathbb{R}_+}$ is a deterministic function, i.e. $(Z_t)_{t\in\mathbb{R}_+}$ is written as in (7). In this case $i_t D_t$ is still a derivation operator, and we have the product rule

$$D_t(FG) = FD_t G + GD_t F + \phi_t D_t F D_t G, \quad t \in \mathbb{R}_+, \tag{11}$$

cf. Prop. 1.3 of [11]. In fact D_t can be written as

$$D_t = j_t \frac{1}{\phi_t}\Delta_t^\phi + i_t D_t,$$

where Δ_t^ϕ is the finite difference operator defined on random functionals by addition at time t of a jump of height ϕ_t to $(Z_t)_{t\in\mathbb{R}_+}$. If $\phi_t \neq 0$, this implies

$$D_t e^F = \frac{e^F}{\phi_t}(e^{\phi_t D_t F} - 1),$$

which converges to $e^F D_t F$ as $\phi_t \to 0$. The following proposition extends Cor. 2.2 of [14] and Th. 2.1 of [13], which are valid for $\phi_t = 1$, $t \in \mathbb{R}_+$. It can also be viewed as a tensorisation of logarithmic Sobolev inequalities for independent Brownian and Poisson processes.

Corollary 3 *Let $F \in \mathrm{Dom}(D)$ be bounded and $\mathcal{F}_T$-measurable, with $F > \eta$ for some $\eta > 0$. We have*

$$\mathrm{Ent}_\pi[F] \leq \frac{1}{2}E_\pi\left[\int_0^T (2 - i_t)D_t F D_t \log F dt\right]. \tag{12}$$

Proof. We apply Cor. 2 and the relation $\phi_t D_t e^F = e^F(e^{\phi_t D_t F} - 1)$ which shows that for positive F,

$$\phi_t D_t \log F = \log(F + \phi_t D_t F) - \log F.$$

□

The following corollary is the analog of the sharp inequality Cor. 5.8 of [8]. For $\phi_t = 1$, $t \in \mathbb{R}_+$, it coincides with Th. 3.4 of [13] and Cor. 2.3 of [14].

Corollary 4 *Let $F \in \mathrm{Dom}(D)$ be bounded and $\mathcal{F}_T$-measurable, with $F > \eta$ for some $\eta > 0$. We have*

$$\mathrm{Ent}_\pi[e^F] \leq E_\pi \left[e^F \int_0^T j_t \frac{1}{\phi_t^2} (\phi_t D_t F e^{\phi_t D_t F} - e^{\phi_t D_t F} + 1) dt + \frac{e^F}{2} \int_0^T i_t |D_t F|^2 dt \right]. \tag{13}$$

Proof. We use the relations $F + \phi_t D_t F = \log(e^F + \phi_t D_t e^F)$ and $e^F + \phi_t D_t e^F = e^F e^{\phi_t D_t F}$:

$$\begin{aligned}
\Psi(e^F, \phi_t D_t e^F) &= (e^F + \phi_t D_t e^F) \log(e^F + \phi_t D_t e^F) - F e^F - \phi_t (1+F) D_t e^F \\
&= e^F e^{\phi_t D_t F}(F + \phi_t D_t F) - F e^F - (1+F) e^F (e^{\phi_t D_t F} - 1) \\
&= e^F (\phi_t D_t F e^{\phi_t D_t F} - e^{\phi_t D_t F} + 1),
\end{aligned}$$

and apply Prop. 1. □

In Cor. 4 the limit of the term in ϕ_t

$$e^F \int_0^T \frac{1}{\phi_t^2} (\phi_t D_t F e^{\phi_t D_t F} - e^{\phi_t D_t F} + 1) dt$$

as ϕ_t tends to zero is exactly the term in i_t: $e^F \frac{1}{2} \int_0^T i_t \frac{1}{F} |D_t F|^2 dt$. If $\phi_t = 0$, i.e. $i_t = 1$, $t \in \mathbb{R}_+$, then $(M_t)_{t \in \mathbb{R}_+}$ is a Brownian motion and from Cor. 1 we obtain the classical modified Sobolev inequality

$$\mathrm{Ent}_\pi[F] \leq \frac{1}{2} E_\pi \left[\frac{1}{F} \|DF\|^2_{L^2([0,T])} \right]. \tag{14}$$

If $\phi_t = 1$, $t \in \mathbb{R}_+$ then $i_t = 0$, $t \in \mathbb{R}_+$, $(M_t)_{t \in \mathbb{R}_+}$ is a standard compensated Poisson process and from Cor. 1 we obtain the modified Sobolev inequality of [1], [2]:

$$\mathrm{Ent}_\pi[F] \leq E_\pi \left[\frac{1}{F} \|DF\|^2_{L^2([0,T])} \right]. \tag{15}$$

Remark *a*) It is known that D_t is a derivation only in the Brownian case, cf. [9], [12], hence only in this case can the modified Sobolev inequality (14) be transformed into the standard Sobolev inequality $\mathrm{Ent}_\pi[F^2] \leq 2E_\pi[\|DF\|^2_{L^2([0,T])}]$ of [7].
b) It follows from Prop. 6 of [12] that for the Azéma martingale, $\phi_t D_t$ is not a finite difference operator, hence (12) and (13) do not hold in this case.

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