Connection, parallel transport, curvature and energy identities on spaces of configurations

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Abstract - *A connection and a covariant derivative are defined in the infinite-dimensional geometry of configuration spaces under Poisson measures. As an application, identities of Weitzenböck type are obtained for anticipating stochastic integrals. This construction relies on the introduction of a particular tangent bundle and associated damped gradient.*

Connexion, transport parallèle, courbure et identités d'énergie sur les espaces de configurations

Résumé - Une connexion et une dérivée covariante sont introduites dans la géométrie de dimension infinie des espaces de configurations sous les mesures de Poisson. Comme application on obtient des identités de type Weitzenböck pour les intégrales stochastiques anticipantes. Cette construction repose sur l'introduction d'un fibré tangent particulier et de son gradient amorti associé, ainsi que sur la compensation de quantités divergentes.

Titre courant: Connection and energy identities on configuration spaces

Version française abrégée

L'espace des trajectoires du mouvement brownien riemannien peut être vu comme une variété de dimension infinie via le calcul des variations stochastique. Des notions de connexion et de courbure ont été introduites dans ce cadre dans [5], voir aussi [1], [6] pour le cas du mouvement brownien à valeurs dans un groupe de Lie pour lequel ce type de connexion est plate. L'espace de Poisson (ou espace des configurations) Γ est un autre exemple d'espace infini-dimensionnel et non linéaire, dont les aspects géométrique ont été étudiés dans [2] en établissant une formule d'intégration par parties. Dans cette Note nous construisons une connexion pour l'espace des configurations à l'aide des méthodes utilisées pour le mouvement brownien à valeurs dans les groupes de Lie. Cette connexion est avec courbure mais sans torsion, ce qui permet de prouver une identité de type Weitzenböck. Le tableau ci-dessous rassemble les correspondances introduites dans cette note entre les géométries de dimension finie et infinie.

Notation	dimension finie	dimension infinie
γ	mesure ponctuelle sur M	élément de Γ
$\mathcal{C}_c^\infty(M)$	fonctions test sur M	vecteurs tangents à Γ
σ	élément de volume de M	métrique riemannienne sur Γ
$\mathcal{U}_c^\infty(M)$	processus stochastiques indexés par M	champs de vecteurs sur Γ

Nous obtenons en particulier des identités en termes d'opérateurs différentiels, du type

$$E\left[\tilde{\delta}(u)^2\right]+E\left[\|d^\Gamma u\|^2_{L^2(\mathbb{R}_+)\wedge L^2(\mathbb{R}_+)}\right]=E\left[\|\nabla^\Gamma u\|^2_{L^2(\mathbb{R}_+)\otimes L^2(\mathbb{R}_+)}\right]+E\left[\|u\|^2_{L^2(\mathbb{R}_+)}\right]$$

si $M=\mathbb{R}_+$, cf. Prop. 3 pour le cas général.

1 Introduction and notation

Let $\Phi(L^2(M))$ denote the Fock space with inner product $\langle\cdot,\cdot\rangle_\Phi$, on a L^2 space $L^2(M,d\sigma)$. Let $D:\Phi(L^2(M))\longrightarrow\Phi(L^2(M))\otimes L^2(M)$ and $\delta:\Phi(L^2(M))\otimes L^2(M)\longrightarrow\Phi(L^2(M))$ denote the unbounded gradient and Skorohod integral operator on $\Phi(L^2(M))$, which are mutually adjoint. An energy identity for δ can be stated as a Weitzenböck type identity, [10]:

$$\|\delta(u)\|^2_\Phi+\frac{1}{2}\int_M\int_M\|D_xu(y)-D_yu(x)\|^2_\Phi\sigma(dx)\sigma(dy)=\|u\|^2_{\Phi\otimes L^2(M)}+\|Du\|^2_{\Phi\otimes L^2(M^2)}. \tag{1}$$

This identity makes sense on flat Wiener space via the Wiener-Itô isomorphism. On the path space of a Riemannian manifold, notions of connection and curvature have been introduced in [5], see also [1] on loop groups. These constructions have led to other Weitzenböck type identities in terms of the intrinsic gradient and divergence on path space, cf. [4], [6], [7]. The proof of (1) being dependent only on the Fock structure, it also makes sense on Poisson space via the Wiener-Poisson isomorphism, which identifies D to a finite difference operator and δ to a compensated Poisson stochastic integral on deterministic functions. Nevertheless, (1) is not of interest here since as in the Brownian case, we are searching for isometry formulas that involve intrinsic differential operators on Γ. For this we need a differentiable structure on M, which is assumed to be a Riemannian manifold. Let ∇^M and div^M denote the gradient and divergence on M, let T_xM denote the tangent space at $x\in M$, and assume that σ is the volume element of M, under which div^M and ∇^M are adjoint: $\langle\mathrm{div}^MU,v\rangle_{L^2(M)}=\langle U,\nabla^Mv\rangle_{L^2(M;\mathrm{T}M)}$, $v\in\mathcal{C}_c^\infty(M)$, $U\in\mathcal{C}_0^\infty(M;\mathrm{T}M)$, where $\mathcal{C}_0^\infty(M;\mathrm{T}M)$ is a space of $\mathcal{C}_b^\infty$ vector fields satisfying suitable boundary conditions. Let $\Gamma=\left\{\gamma=\sum_{i=1}^{i=n}\epsilon_{x_i}\ :\ (x_i)_{i=1}^{i=n}\subset M,\ \ x_i\neq x_j\ \forall i\neq j,\ n\in\mathbb{N}\cup\{\infty\}\right\}$, denote the configuration space on M, where ϵ_x denotes the Dirac measure at $x\in M$, with the vague topology and associated σ-algebra, cf. [2]. We identify $\gamma=\sum_{i=1}^{i=n}\epsilon_{x_i}$ with the locally finite configuration $\{x_i\ :\ 0\leq i\leq n\}$. We assume that σ is a Radon measure, and let P denote the Poisson measure with intensity σ on Γ. We consider a space $\mathcal{S}$ of cylindrical functionals $F:\Gamma\longrightarrow\mathbb{R}$ for which there is a compact set K and a family $(f_n)_{n\in\mathbb{N}}\subset\mathcal{C}_c^\infty(K)$ of symmetric functions such that card $(\gamma\cap K)=n$ implies

$$F(\gamma)=f_n(x_1,\dots,x_n),\qquad\gamma=\{x_1,\dots,x_n\}.$$

Let $\mathcal{U}_c^\infty(M)$ denote the space of smooth processes of the form

$$u(\gamma, x) = \sum_{i=1}^{i=n} F_i(\gamma) h_i(x), \quad (\gamma, x) \in \Gamma \times M, \quad h_i \in \mathcal{C}_c^\infty(M), \quad F_i \in \mathcal{S}, \; i = 1, \ldots, n. \tag{2}$$

2 Damped gradient and tangent bundle

Let $\mathrm{Diff}(M)$ denote the group of diffeomorphisms of M, let $V(M) = \mathcal{C}^\infty(M; \mathrm{T}M)$ be the Lie algebra of $\mathcal{C}^\infty$ vector fields on M, let $(\phi_t^U)_{t \in \mathrm{R}_+}$ denote the flow generated by the vector field $U \in V(M)$, let $\phi_t^U(\gamma)$ denote the image measure of $\gamma \in \Gamma$ by $\phi_t^U : M \longrightarrow M$, and let $\hat{D}$ be the intrinsic gradient operator defined on $\mathcal{S}$ as

$$\hat{D}_x F(\gamma) = \sum_{i=1}^{i=n} 1_{\{x_i\}}(x) \nabla_i^M f_n(x_1, \ldots, x_n), \quad \gamma(dx) - a.e., \quad \gamma = \{x_1, \ldots, x_n\}, \quad n \geq 1,$$

or

$$\hat{D}_U(\gamma) F = \langle \hat{D} F, U \rangle_{L^2(M, \mathrm{T}M, d\gamma)} = \lim_{\varepsilon \to 0} \frac{F(\phi_\varepsilon^U(\gamma)) - F(\gamma)}{\varepsilon}, \quad U \in V(M),$$

cf. [2]. This definition suggests to consider $L^2(M, \mathrm{T}M, d\gamma)$ as the tangent space to $\gamma \in \Gamma$ (see Sect. 3 and Def. 3.2 of [2]). In this note we make a completely different choice that allows to formulate Weitzenböck type identities. We define a trivial tangent bundle to Γ with fiber $\mathcal{C}_c^\infty(M)$ as $\mathrm{T}\Gamma = \Gamma \times \mathcal{C}_c^\infty(M)$, with group action

$$\begin{aligned} \mathrm{T}\Gamma \times \mathrm{Diff}(M) &\longrightarrow \mathrm{T}\Gamma \\ (\gamma, u, \phi) &\mapsto (\gamma, u \circ \phi). \end{aligned}$$

A stochastic process $u \in \mathcal{U}_c^\infty(M)$ is identified to the smooth vector field $\gamma \mapsto (\gamma, u(\gamma, \cdot)) \in \mathrm{T}\Gamma$. We need to define a damped gradient, cf. [8], and its associated tangent bundle. Let $g(x, y)$ denote the Green kernel on $M \times M$ associated to the Laplacian $\mathcal{L} = \mathrm{div}^M \nabla^M$ on M, and let

$$\partial_x(y) = \nabla_x^M g(x, y) \in T_x M, \quad \sigma \otimes \sigma(dx, dy) - a.e.,$$

and

$$\tilde{u}(x) = \nabla^M \mathcal{L}^{-1} u(x) = \int_M u(y) \partial_x(y) \sigma(dy) \in \mathrm{T}_x M, \quad x \in M, \; u \in \mathcal{U}_c^\infty(M).$$

We have $\mathrm{div}^M \tilde{u} = u$ and $\langle \nabla^M v, \tilde{u} \rangle_{L^2(M; \mathrm{T}M, \sigma)} = \langle u, v \rangle_{L^2(M, \sigma)}$, $u, v \in \mathcal{C}_c^\infty(M)$.

Definition 1 *The damped gradient $\tilde{D} : L^2(\Gamma) \longrightarrow L^2(\Gamma; L^1(M))$ is defined on $\mathcal{S}$ as*

$$\tilde{D}_y F(\gamma) = \sum_{i=1}^{i=n} \partial_{x_i}(y) f_n(x_1, \ldots, x_n), \quad \sigma(dy) - a.e., \quad \gamma = \{x_1, \ldots, x_n\} \in \Gamma, \quad n \geq 1. \tag{3}$$

The notation $\partial_{x_i}(y) f_n(x_1, \ldots, x_n)$ denotes the application of the derivation $\partial_{x_i}(y)$ to the i-th variable of f_n. We also have

$$\langle \tilde{D}F(\gamma), u\rangle_{L^2(M,d\sigma)} = \lim_{\varepsilon\to 0} \frac{F(\phi_\varepsilon^{\tilde{u}}(\gamma)) - F(\gamma)}{\varepsilon}, \quad u \in \mathcal{C}_c^\infty(M), \quad F \in \mathcal{S},$$

and let $\tilde{D}_u F = \langle \tilde{D}F, u\rangle_{L^2(M,d\sigma)}$, $u \in \mathcal{U}_c^\infty(M)$. The relation $\widetilde{\mathrm{div}^M} U = U$, $U \in \mathcal{C}_0^\infty(M; \mathrm{T}M)$, holds only for $M = \mathbb{R}_+$ and $\mathcal{C}_0^\infty(\mathbb{R}_+; \mathbb{R}) = \{u \in \mathcal{C}_b^\infty(\mathbb{R}_+; \mathbb{R}) \ : \ u(0) = 0\}$. Similarly, the relation $\hat{D}_U F = \tilde{D}_{\mathrm{div}^M U} F$ holds for $M = \mathbb{R}_+$ and in the general case we only have $E[\hat{D}_U F] = E[\tilde{D}_{\mathrm{div}^M U} F]$, $F \in \mathcal{S}$, $U \in \mathcal{C}_0^\infty(M; \mathrm{T}M)$. We have

$$\tilde{D}_u F = \langle DF, u\rangle_{L^2(M,d\sigma)} + \delta(\tilde{u} DF), \quad F \in \mathcal{S}, \tag{4}$$

where $D_x(F)(\gamma) = F(\gamma \cup \{x\}) - F(\gamma)$, $\sigma(dx)$-a.e., $\gamma \in \Gamma^M$, is the gradient on Fock space, cf. Remark 3 of [9] and Th. 8.2.1 of [8]. The operator $\tilde{D}$ has a closable adjoint (divergence) $\tilde{\delta}$ such that $E[F\tilde{\delta}(u)] = E[\langle \tilde{D}F, u\rangle_{L^2(M,d\sigma)}]$, $F \in \mathcal{S}$, $u \in \mathcal{U}_c^\infty(M)$, and $\tilde{\delta}(h) = \delta(h) = \int_M h(d\gamma - d\sigma)$, $h \in \mathcal{C}_c^\infty(M)$.

3 Connection, covariant derivative and Lie-Poisson bracket

Definition 2 *The covariant derivative $\nabla_v^\Gamma u$ of $u \in \mathcal{U}_c^\infty(M)$ in the direction $v \in \mathcal{U}_c^\infty(M)$ is defined for u as in (2) by:*

$$\nabla_v^\Gamma u(x) = \sum_{i=1}^{i=n} \left[h_i(x) \tilde{D}_v F_i + F_i \langle \tilde{v}(x), \nabla^M h_i(x)\rangle_{\mathrm{T}_x M} \right], \quad x \in M.$$

This definition has an interpretation in terms of a decomposition of the tangent space to $\mathrm{T}\Gamma$ at $u \in \mathrm{T}\Gamma$ in horizontal and vertical subspaces as $Q \oplus V(M)$, where $Q = \{(v, -\tilde{v}) \ : \ v \in \mathcal{C}_c^\infty(M)\}$, i.e. $(v, U) = (v, -\tilde{v}) \oplus (0, U + \tilde{v})$, $v \in \mathcal{C}_c^\infty(M)$, $U \in V(M)$. The horizontal lift starting from $(\gamma, u) \in \mathrm{T}\Gamma$ of the curve $t \mapsto \phi_t^{\tilde{v}}(\gamma)$ is $t \mapsto (\phi_t^{\tilde{v}}(\gamma), u \circ \phi_{-t}^{\tilde{v}})$ and the parallel transport $\tau_t^v : \mathrm{T}_{\phi_t^{\tilde{v}}(\gamma)}\Gamma \to \mathrm{T}_\gamma\Gamma$ along $t \mapsto \phi_t^{\tilde{v}}(\gamma)$ is given by $\tau_t^v u = u \circ \phi_t^{\tilde{v}}$, $u \in \mathcal{C}_c^\infty(M)$. Thus $\nabla_v^\Gamma u$ can be obtained as

$$\nabla_v^\Gamma u(\gamma, x) = \lim_{\varepsilon\to 0} \frac{\tau_\varepsilon^v u(\phi_\varepsilon^{\tilde{v}}(\gamma), x) - u(\gamma, x)}{\varepsilon}, \qquad x \in M, \ \gamma \in \Gamma, \ v \in \mathcal{C}_c^\infty(M).$$

We assume that the manifold M is equipped with a Riemannian connection ∇^M without torsion.

Lemma 1 *Let $u, v, w \in \mathcal{C}_c^\infty(M)$. The relation*

$$E[\tilde{D}_w F] = E[(\tilde{D}_u \tilde{D}_v - \tilde{D}_v \tilde{D}_u) F], \quad F \in \mathcal{S}, \tag{5}$$

holds if and only if $w = \nabla_u^\Gamma v - \nabla_v^\Gamma u$.

The Lie bracket $\{u, v\} \in \mathcal{C}_c^\infty(M)$ of $(u, v) \in \mathcal{C}_c^\infty(M) \times \mathcal{C}_c^\infty(M)$ is defined to be the unique $w \in \mathcal{C}_c^\infty(M)$ satisfying (5). It is extended to $u, v \in \mathcal{U}_c^\infty(M)$ by the relation

$$\{Fu, Gv\} = FG\{u, v\} + u\tilde{D}_v F - v\tilde{D}_u G, \quad F, G \in \mathcal{S}.$$

The curvature tensor $\Omega^\Gamma : \mathcal{U}_c^\infty(M) \times \mathcal{U}_c^\infty(M) \times \mathcal{U}_c^\infty(M) \longrightarrow \mathcal{U}_c^\infty(M)$ is defined as

$$\Omega^\Gamma(u, v)h = [\nabla_u^\Gamma, \nabla_v^\Gamma]h - \nabla_{\{u,v\}}^\Gamma h, \qquad u, v, h \in \mathcal{U}_c^\infty(M).$$

The connection defined by ∇^Γ has a vanishing torsion: $\{u, v\} = \nabla_u^\Gamma v - \nabla_v^\Gamma u$, $u, v \in \mathcal{U}_c^\infty(M)$, and it is not Riemannian:

$$\tilde{D}_h \langle u, v\rangle_{L^2(M,d\sigma)} = \langle u, \nabla_h^\Gamma v\rangle_{L^2(M,d\sigma)} + \langle \nabla_h^\Gamma u, v\rangle_{L^2(M,d\sigma)} + \langle u, v\rangle_{L^2(M,hd\sigma)}. \tag{6}$$

Proposition 1 *Let $u, v \in \mathcal{U}_c^\infty(M)$.*

i) If $\Omega^\Gamma(u, v)h = 0$, $h \in \mathcal{C}_c^\infty(M)$, then $\tilde{D}_u\tilde{D}_v - \tilde{D}_v\tilde{D}_u = \tilde{D}_{\{u,v\}}$ on $\mathcal{S}$.

ii) If $M = \mathbb{R}^+$ then $\Omega^\Gamma(u, v) = 0$ on $\mathcal{U}_c^\infty(\mathbb{R}_+)$, $\tilde{D}_u\tilde{D}_v - \tilde{D}_v\tilde{D}_u = \tilde{D}_{\{u,v\}}$ on $\mathcal{S}$, and $\mathcal{U}_c^\infty(\mathbb{R}_+)$ is a Lie algebra under the bracket $\{\cdot, \cdot\}$.

iii) If $u, v \in \mathcal{C}_c^\infty(M)$, then $E[F(\tilde{D}_u\tilde{D}_v - \tilde{D}_v\tilde{D}_u)G] = E[F\tilde{D}_{\{u,v\}}G]$ if F is $\sigma(G)$-measurable, $F, G \in \mathcal{S}$.

4 Weitzenböck type identities

We identify $L^2(M)$ to its dual $L^2(M)^*$ via the scalar product, and let $L^2(M)\wedge L^2(M)$ denote the space of continuous antisymmetric bilinear forms on $L^2(M) \otimes L^2(M)$.

Definition 3 *The coboundary $d^\Gamma u$ of $u \in \mathcal{U}_c^\infty(M)$ is defined as*

$$\langle d^\Gamma u, h_1 \wedge h_2\rangle_{L^2(M)\wedge L^2(M)} = \langle \nabla_{h_1}^\Gamma u, h_2\rangle_{L^2(M)} - \langle \nabla_{h_2}^\Gamma u, h_1\rangle_{L^2(M)}, \quad h_1, h_2 \in \mathcal{U}_c^\infty(M).$$

We also let $\nabla_y^\Gamma v(x) = \partial_x(y)v(x) + \tilde{D}_y v(x)$, $x, y \in M$, i.e. $\nabla_u^\Gamma v(x) = \int_M u(y)\nabla_y^\Gamma v(x)\sigma(dy)$, $x \in M$, $u, v \in \mathcal{U}_c^\infty(M)$. We start by considering $M = \mathbb{R}_+$ with σ the Lebesgue measure, i.e. $g(x, y) = -x \vee y$, $\partial_x(y) = -1_{[0,x]}(y)$, and $\tilde{u}(x) = -\int_0^x u(t)dt$, $x, y \in \mathbb{R}_+$, $u \in \mathcal{C}_c^\infty(\mathbb{R}_+)$. Then $\tilde{D}$ is the gradient of [3] since from (3) we have

$$\tilde{D}_x F(\gamma) = -\sum_{i=1}^{i=n} 1_{[0,x_k]}(x)\partial_i f_n(x_1, \ldots, x_n), \quad x \in \mathbb{R}_+,\ \gamma = \{x_1, \ldots, x_n\} \in \Gamma.$$

Proposition 2 *If $M = \mathbb{R}_+$, we have the Weitzenböck type identity for $u \in \mathcal{U}_c^\infty(M)$:*

$$E\left[\tilde{\delta}(u)^2\right] + E\left[\|d^\Gamma u\|^2_{L^2(\mathbb{R}_+)\wedge L^2(\mathbb{R}_+)}\right] = E\left[\|\nabla^\Gamma u\|^2_{L^2(\mathbb{R}_+)\otimes L^2(\mathbb{R}_+)}\right] + E\left[\|u\|^2_{L^2(\mathbb{R}_+)}\right].$$

We deduce the following bound for the anticipating stochastic integral operator $\tilde{\delta}$:

$$E\left[\tilde{\delta}(u)^2\right] \leq E\left[\|u\|^2_{L^2(\mathrm{R}_+)}\right] + E\left[\|\nabla^\Gamma u\|^2_{L^2(\mathrm{R}_+)\otimes L^2(\mathrm{R}_+)}\right], \quad u \in \mathcal{U}_c^\infty(\mathbb{R}_+).$$

The de Rham-Hodge-Kodaira operator $\Box = (d^\Gamma)^* d^\Gamma + \tilde{D}\tilde{\delta}$ can be written as $\Box = \nabla^*\nabla + I_d$ on $\mathcal{U}_c^\infty(\mathbb{R}_+)$. In the general case we have the following.

Proposition 3 *Let $u \in \mathcal{U}_c^\infty(M)$ be a process of the form $u = \sum_{i=1}^{i=n} u_i F_i$, such that for $i, j = 1, \ldots, n$, either i) F_i is $\sigma(F_j)$-measurable, or ii) $\Omega^\Gamma(u_i, v_j) = 0$ on $\mathcal{U}_c^\infty(U)$. Then*

$$\begin{aligned} & E[\tilde{\delta}(u)^2] + E\left[\int_M \int_M \tilde{D}_y u(x) \tilde{D}_x u(y) \sigma(dy)\sigma(dx)\right] \\ &= E[\|u\|^2_{L^2(M, d\sigma)}] + 2E\left[\int_M \int_M \nabla^\Gamma_y u(x) \tilde{D}_x u(y) \sigma(dy)\sigma(dx)\right]. \end{aligned} \tag{7}$$

Proof. As in Th. 4.3 of [4] or Th. 3.3. of [6], we use (4), applied to $F = \tilde{\delta}(h)$, $h \in \mathcal{C}_c^\infty(M)$, and the relations $E[F_i(\tilde{D}_{u_i}\tilde{D}_{v_j} - \tilde{D}_{v_j}\tilde{D}_{u_i})G_j] = E[F_i \tilde{D}_{\{u_i, v_j\}} G_j]$ and $\tilde{D}_{u_i}\tilde{D}_{v_j} - \tilde{D}_{v_j}\tilde{D}_{u_i} = \tilde{D}_{\{u_i, v_j\}}$, which hold respectively if *i*) or *ii*) is satisfied. $\Box$

If dim $M = 1$ we have

$$\begin{aligned} & \int_M \int_M \nabla^\Gamma_y u(x) \nabla^\Gamma_x u(y) \sigma(dy)\sigma(dx) - \int_M \int_M \partial_x(y) u(x) \partial_y(x) u(y) \sigma(dy)\sigma(dx) \\ &= 2\int_M \int_M \nabla^\Gamma_y u(x) \tilde{D}_x u(y) \sigma(dy)\sigma(dx) - \int_M \int_M \tilde{D}_y u(x) \tilde{D}_x u(y) \sigma(dy)\sigma(dx), \end{aligned} \tag{8}$$

hence (7) can be rewritten as

$$\begin{aligned} & E[\tilde{\delta}(u)^2] + E\left[\|d^\Gamma u\|^2_{L^2(M)\wedge L^2(M)}\right] + E\left[\int_M \int_M \partial_x(y) u(x) \partial_y(x) u(y) \sigma(dy)\sigma(dx)\right] \\ &= E[\|u\|^2_{L^2(M)}] + E\left[\|\nabla^\Gamma u\|^2_{L^2(M)\otimes L^2(M)}\right], \quad u \in \mathcal{U}_c^\infty(M), \end{aligned} \tag{9}$$

and Prop. 2 follows as a particular case from Prop. 1-ii), Prop. 3 and the relation $\partial_x(y)\partial_y(x) = 0$ for a.e. $x, y \in M = \mathbb{R}_+$. If dim $M > 1$ however, (8) becomes an equality between finite terms that are cancellations of infinite terms since we only have $g(x, \cdot) \in W^{1,1}(M; TM)$, and (9) should be interpreted accordingly.

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