Quasi-invariance for Lévy processes under anticipating shifts

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Abstract

We prove a Girsanov theorem for the combination of a Brownian motion on $\mathbb{R}_+$ and a Poisson random measure on $\mathbb{R}_+ \times [-1,1]^d$ under random anticipating transformations of paths and configurations. The factorization of the density function via Carleman-Freholm determinants and divergence operators appears as an extension of the martingale factorization in the adapted jump case.

Key words: Quasi invariance, Lévy processes, Poisson random measures.
Mathematics Subject Classification (1991). Primary: 60B11, 60H07, 60G15, 60G57. Secondary: 28C20, 46G12.

1 Introduction

The Cameron-Martin theorem [5] gives the density with respect to the Wiener measure of a deterministic shift of Brownian motion. Similarly, the Skorokhod theorem on invariance of measures [18] gives the density with respect to Poisson measures of deterministic shifts of configuration. These theorems have an extension (the Girsanov theorem) to random shifts under adaptedness hypothesis. Given a martingale $(\tilde{M}(t))_{t\in\mathbb{R}_+}$ on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\mathbb{R}_+}, P)$ and a probability Q absolutely continuous with respect to P, the classical Girsanov theorem, [7], [8], gives a canonical decomposition of $(\tilde{M}(t))_{t\in\mathbb{R}_+}$ as a sum of a continuous martingale $(M^c(t))_{t\in\mathbb{R}_+}$ under Q, a pure jump martingale $(M^d(t))_{t\in\mathbb{R}_+}$ under Q and a finite variation process. Since $(M^c(t))_{t\in\mathbb{R}_+}$ and $(M^d(t))_{t\in\mathbb{R}_+}$ are obtained by shifts of the values of $(\tilde{M}(t))_{t\in\mathbb{R}_+}$, the Girsanov theorem also allows to compute the density with respect to P of a transformation of the space Ω. Thus there is a strong analogy between the Girsanov theorem and change of variable formulas in the theory of integration, the density of P with respect to Q being computed with a Jacobian determinant. The classical Girsanov theorem relies on Itô's stochastic calculus, and in particular on adaptedness hypothesis which are not needed in integration (change of variable formula) techniques. The latter point of view has proved to be useful to remove the adaptedness conditions imposed by the Girsanov theorem on transformations of trajectories. In order to deal with stochastic processes, a theory of integration in infinite dimensions is needed, and analysis on the Wiener space provides such a framework in the case of Brownian motion. The extension of the Girsanov theorem to anticipating shifts of Brownian motion has been carried out in [10], [17], [19], [20], see the book [21] for more complete references. In the standard Poisson case (i.e. for Poisson random measures based on $\mathbb{R}_+$), its analog has been treated in [14], [16], using analysis for an infinite product of exponential densities. This result relies on the interpretation of Poisson samples on $\mathbb{R}_+$ as sequences of independent exponentially distributed interjump distances. On the other hand, a change of variable formula for the uniform density in infinite dimensions has been established in [15].

In this paper we obtain a Girsanov type theorem for random shifts of a Poisson random measure on $\mathbb{R}_+ \times [-1, 1]^d$ and a Brownian motion, i.e. an anticipative Girsanov theorem for Lévy processes. The main observation is that a Poisson random random measure on $\mathbb{R}_+ \times [-1, 1]^d$ with flat intensity consists in randomly distributed sets of points (configurations) that can be represented as sequences of independent $d+1$-dimensional random variables whose first component is exponentially distributed, the remaining independent d components having uniform laws on $[-1, 1]$. The Radon-Nikodym density function is then factorized with a divergence operator and a Carleman-Fredholm determinant and we allow for interactions between the different components of the process. This factorization of the density is similar to the expression of the density via stochastic calculus, as the solution of a stochastic differential equation. Girsanov type theorems for non-adapted shifts of Poisson random measures are completely natural since in the Poisson case on $\mathbb{R}^{d+1}$ there is no canonical notion of time or filtration.

We proceed as follows. In Sect. 2 we review different versions of the adapted Girsanov theorem. In Sect. 3 we introduce some notation, in particular an interpretation of the Poisson space as a space of sequences. The main results are stated in Sect. 4, for shifts of configuration points that are expressed as perturbations of interjump times and jump heights. In Sect. 5 and Sect. 6 we prove technical results and then our extension of the Girsanov theorem to the anticipating case.

2 Adapted Girsanov theorem and change of variable formulas

In this section we review the classical Skorokhod theorem on the absolute continuity of Poisson measures under deterministic diffeomorphisms, cf. [1], [18], [22], and its extension to adapted shifts, i.e. the Girsanov theorem, cf. [7], [8]. In Sect. 4 we will present an extension of this theorem to the anticipating case.

Let $\Gamma(\mathbb{R}_+ \times [-1,1]^d)$ denote the configuration space on $\mathbb{R}_+ \times [-1,1]^d$, i.e. the set of Radon measures on $\mathbb{R}_+ \times [-1,1]^d$ of the form

$$\sum_{k=1}^{k=N} \varepsilon_{x_k}, \quad (x_k)_{k=1}^{k=N} \subset \mathbb{R}_+ \times [-1,1]^d, \ x_k \neq x_l, \ \forall k \neq l, \ N \in \mathbb{N} \cup \{\infty\}.$$

A configuration γ is a sum of Dirac measures ε_y and will be identified to the discrete set of points that defines its support, in particular we will write $(s,y) \in \gamma$ whenever $\gamma(\{(s,y)\}) = 1$. Let $\mathcal{C}_0(\mathbb{R}_+)$ denote the space of continuous functions starting at 0. Let $\Omega = \mathcal{C}_0(\mathbb{R}_+) \times \Gamma(\mathbb{R}_+ \times [-1,1]^d)$ and consider a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\mathbb{R}_+}, P)$. Let ν be a diffuse random measure on $\mathbb{R}_+ \times [-1,1]^d$, bounded on compact sets, $\mathcal{F}_t$-predictable under P, i.e. $t \mapsto \int_0^t \int_{[-1,1]^d} u(s,y)\nu(ds,dy)$ is $\mathcal{F}_t$-predictable for every positive bounded $\mathcal{F}_t$-predictable process $(u(s,y))_{(s,y)\in\mathbb{R}_+\times[-1,1]^d}$. We assume that the canonical continuous process $t \mapsto \omega(t)$ defined on $\mathcal{C}_0(\mathbb{R}_+)$ has an $\mathcal{F}_t$-predictable quadratic variation $(\beta(t))_{t\in\mathbb{R}_+}$ under P, and that ν is the intensity (or Lévy measure, or dual predictable projection) of the random measure $\mu : \Omega \longrightarrow \Gamma(\mathbb{R}_+ \times [-1,1]^d)$ defined as $\mu(\omega,\gamma) = \gamma$, i.e. $\nu(ds,dy)$ satisfies

$$E_P\left[\int_{\mathbb{R}_+\times[-1,1]^d} u(s,y)\gamma(ds,dy)\right] = E_P\left[\int_{\mathbb{R}_+\times[-1,1]^d} u(s,y)\nu(ds,dy)\right],$$

for every positive bounded $\mathcal{F}_t$-predictable process $(u(s,y))_{(s,y)\in\mathbb{R}_+\times[-1,1]^d}$. Let $\phi : \mathbb{R}_+ \times [-1,1]^d \longrightarrow \mathbb{R}_+ \times [-1,1]^d$ be a random diffeomorphism of $\mathbb{R}_+ \times [-1,1]^d$ whose $d+1$ components

$$(\phi^1(t,y))_{(t,y)\in\mathbb{R}_+\times[-1,1]^d}, \ldots, (\phi^{d+1}(t,y))_{(t,y)\in\mathbb{R}_+\times[-1,1]^d},$$

are $\mathcal{F}_t$-predictable processes. We assume that there exists a predictable process $(Z(s,y))_{(s,y)\in\mathbb{R}_+\times[-1,1]}$ -1, P-a.s., such that

(i) $Z \circ \phi \in L^1(\mathbb{R}_+ \times [-1,1]^d, \nu)$, i.e.

$$\int_{\mathbb{R}_+\times[-1,1]^d} |Z \circ \phi(s,y)|\nu(ds,dy) < \infty, \quad P-a.s,$$

(ii) and

$$\int_{\mathbb{R}_+\times[-1,1]^d} g(\phi(s,y))(1+Z\circ\phi(s,y))\nu(ds,dy) = \int_{\mathbb{R}_+\times[-1,1]^d} g(s,y)\nu(ds,dy), \quad (2.1)$$

$\forall g \in \mathcal{C}_c^+(\mathbb{R}^{d+2})$, i.e. $(1+Z)^{-1}$ is the density of $\phi_*\nu$ with respect to ν, P-a.s.

If ν is a multiple of the Lebesgue measure, then $1 + Z \circ \phi(s,y) = |\partial\phi(s,y)|$ is the Jacobian determinant of ϕ. Let $\phi(\gamma)$ denote the configuration γ whose points are shifted according to ϕ, i.e. $\phi(\gamma)$ is identified to the set

$$\phi(\gamma) = \{\phi((s,y)) \ : \ (s,y) \in \gamma\}, \qquad \gamma \in \Gamma(\mathbb{R}_+ \times [-1,1]^d).$$

Let $(z(s))_{s\in\mathbb{R}_+} \in L^2(\Omega)\otimes L^2(\mathbb{R}_+, d\beta)$ be a square-integrable $\mathcal{F}_t$-predictable process and let $\psi(\omega)$ be defined as

$$\psi(\omega)(t) = \omega(t) + \int_0^t z(s)d\beta(s), \quad t \in \mathbb{R}_+, \ \omega \in \mathcal{C}_0(\mathbb{R}_+).$$

In the following result, $(M^c(t))_{t\in\mathbb{R}_+}$ is a time changed Brownian motion and μ is a Poisson random measure with deterministic intensity ν.

Theorem 2.1 *Assume that ν and β are deterministic, and*

$$E_P\left[\exp\left(-\int_0^\infty z(s)d\omega(s) - \int_{\mathbb{R}^d} Z\circ\phi(s,y)\nu(ds,dy) - \frac{1}{2}\int_0^\infty z^2(s)d\beta(s)\right)\right.$$
$$\left.\times \prod_{(s,y)\in\gamma}(1+Z\circ\phi(s,y))\right] = 1. \quad (2.2)$$

Then for every bounded and measurable random variable $f : \Omega \longrightarrow \mathbb{R}$ we have

$$\begin{aligned} E_P[f(\omega,\gamma)] &= E_P\left[f(\psi(\omega),\phi(\gamma))\exp\left(-\int_0^\infty z(s)d\omega(s) - \int_{\mathbb{R}^d} Z\circ\phi(s,y)\nu(ds,dy)\right.\right. \\ &\left.\left. -\frac{1}{2}\int_0^\infty z^2(s)d\beta(s)\right)\prod_{(s,y)\in\gamma}(1+Z\circ\phi(s,y))\right]. \end{aligned}$$

We can also write

$$\begin{aligned} \frac{d\Phi_*^{-1}P}{dP} &= \exp\left(-\int_0^\infty z(s)d\omega(s) + \int_{\mathbb{R}^d} Z\circ\phi(s,y)\nu(ds,dy) - \frac{1}{2}\int_0^t z^2(s)d\beta(s)\right) \\ &\times \prod_{(s,y)\in\gamma}(1+Z\circ\phi(s,y)), \end{aligned}$$

where the transformation $\Phi : \Omega \longrightarrow \Omega$ is defined as $\Phi(\omega,\gamma) = (\psi(\omega),\phi(\gamma))$, $(\omega,\gamma) \in \Omega$. We will recall a proof of the above result using an extension of the classical Girsanov theorem to the jump case in the martingale framework, cf. [8], [9]:

Theorem 2.2 *Let $(\tilde{M}(t))_{t\in\mathbb{R}_+}$ be a martingale on $(\Omega, \mathcal{F}, P)$, whose continuous part $(\tilde{M}^c(t))_{t\in\mathbb{R}_+}$ has quadratic variation $(\tilde{\beta}(t))_{t\in\mathbb{R}_+}$. Assume that the jump part $(\tilde{M}^d(t))_{t\in\mathbb{R}_+}$ of $(\tilde{M}(t))_{t\in\mathbb{R}_+}$ is given by the discrete random measure $\tilde{\mu} : \Omega \longrightarrow \Gamma(\mathbb{R}_+ \times [-1,1]^d)$ with intensity $\tilde{\nu}$ as*

$$\tilde{M}^d(t) = \int_0^t \int_{[-1,1]^d} \tilde{\mu}(ds, dy) - \int_0^t \int_{[-1,1]^d} \tilde{\nu}(ds, dy), \quad t \in \mathbb{R}_+.$$

Let

$$(\tilde{Z}(s,y))_{(s,y)\in\mathbb{R}_+\times[-1,1]^d} \in L^1(\mathbb{R}_+ \times [-1,1]^d, \tilde{\nu}), \quad \text{with } \tilde{Z} > -1, \quad P - a.s.,$$

and $(\tilde{z}(s))_{s\in\mathbb{R}_+} \in L^2(\Omega) \otimes L^2(\mathbb{R}_+, d\tilde{\beta})$ be predictable processes, and define the measure Q by its density

$$\begin{aligned}\frac{dQ}{dP} &= \exp\left(-\int_0^\infty \tilde{z}(s)d\tilde{M}^c(s) - \int_{\mathbb{R}_+\times[-1,1]^d} \tilde{Z}(s,y)\tilde{\nu}(ds,dy) - \frac{1}{2}\int_0^\infty \tilde{z}^2(s)d\tilde{\beta}(s)\right) \\ &\quad \times \prod_{(s,y)} (1 + \tilde{Z}(s,y)\Delta\tilde{M}(s,y)),\end{aligned} \tag{2.3}$$

where $\Delta\tilde{M}(s,y) = \tilde{\mu}(\{(s,y)\})$, $(s,y) \in \mathbb{R}_+ \times [-1,1]^d$.
If Q is a probability measure then, under Q, the process $(\tilde{M}(t))_{t\in\mathbb{R}_+}$ admits a unique decomposition

$$\tilde{M}(t) = M^c(t) + M^d(t) + \alpha(t), \quad t \in \mathbb{R}_+,$$

where

(i) $M^c(t) = \tilde{M}^c(t) + \int_0^t \tilde{z}(s)d\beta(s)$, $t \in \mathbb{R}_+$, is a continuous local martingale under Q with predictable quadratic variation $\beta^c(t) = \tilde{\beta}^c(t)$, $t \in \mathbb{R}_+$,

(ii) $M^d(t) = \tilde{M}^d(t) - \int_0^t \int_{[-1,1]^d} \tilde{Z}(s,y)\tilde{\nu}(ds,dy)$, $t \in \mathbb{R}_+$, is a pure jump martingale and $d\tilde{\mu}$ has intensity $(1+\tilde{Z})d\tilde{\nu}$ under Q,

(iii) $\alpha(t) = \int_0^t \int_{[-1,1]^d} \tilde{Z}(s,y)\tilde{\nu}(ds,dy) - \int_0^t \tilde{z}(s)d\beta(s)$, $t \in \mathbb{R}_+$, is a finite variation process.

Proof. of Th. 2.1 from Th. 2.2. let $\tilde{Z} = Z$, $\tilde{z} = z$, assume that $(\tilde{M}(t))_{t\in\mathbb{R}_+} = (\tilde{M}^d(t))_{t\in\mathbb{R}_+} + (\tilde{M}^c(t))_{t\in\mathbb{R}_+}$ is defined as $\tilde{M}^c(t) = \omega(t)$ and

$$\tilde{M}^d(t) = \int_0^t \int_{[-1,1]^d} \phi(\gamma)(ds,dy) - \int_0^t \int_{[-1,1]^d} \tilde{\nu}(ds,dy), \quad t \in \mathbb{R}_+,$$

with $\tilde{\mu}(\omega,\gamma) = \phi(\gamma)$ and $\tilde{\nu}(ds,dy) = (1+Z(s,y))^{-1}\nu(ds,dy)$. Let $(\tilde{\mathcal{F}}_t)_{t\in\mathbb{R}_+}$ denote the smallest filtration that makes every process $(u(s,y))_{(s,y)\in\mathbb{R}_+\times[-1,1]^d}$ adapted if $(u\circ\phi(s,y))_{(s,y)\in\mathbb{R}_+\times[-1,1]^d}$ is $\mathcal{F}_t$-adapted. The process $(\tilde{M}(t))_{t\in\mathbb{R}_+}$ is $(\tilde{\mathcal{F}}_t)_{t\in\mathbb{R}_+}$ adapted. Given a positive bounded $\tilde{\mathcal{F}}_t$-predictable process, $(u(s,y))_{(s,y)\in\mathbb{R}_+\times[-1,1]^d}$, we have

$$\begin{aligned} E_P\left[\int_{\mathbb{R}_+\times[-1,1]^d} u(s,y)\ \phi(\gamma)(ds,dy)\right] &= E_P\left[\int_{\mathbb{R}_+\times[-1,1]^d} u\circ\phi(s,y)\ \gamma(ds,dy)\right] \\ &= E_P\left[\int_{\mathbb{R}_+\times[-1,1]^d} u\circ\phi(s,y)\ \nu(ds,dy)\right] \\ &= E_P\left[\int_{\mathbb{R}_+\times[-1,1]^d} \frac{u(s,y)}{1+Z(s,y)}\ \nu(ds,dy)\right] \\ &= E_P\left[\int_{\mathbb{R}_+\times[-1,1]^d} u(s,y)\ \tilde{\nu}(ds,dy)\right]. \end{aligned}$$

Hence under P, $\tilde{\mu}$ has intensity $\tilde{\nu}$, and $(\tilde{M}^c(t))_{t\in\mathbb{R}_+}$ has quadratic variation $(\tilde{\beta}(t))_{t\in\mathbb{R}_+} = (\beta(t))_{t\in\mathbb{R}_+}$. Let $(M^c(t))_{t\in\mathbb{R}_+}$ be defined as $M^c(t) = \psi(\omega)(t)$, $t\in\mathbb{R}_+$, let

$$M^d(t) = \int_0^t\int_{[-1,1]^d}\tilde{\mu}(ds,dy) - \int_0^t\int_{[-1,1]^d}\nu(ds,dy).$$

From Th. 2.2, $(M(t))_{t\in\mathbb{R}_+}$ is a martingale under Q, the random measure $\tilde{\mu}$ has intensity ν under Q, and $\psi(\omega)$ is a continuous martingale under Q with quadratic variation $\tilde{\beta}$. Hence $\phi(\gamma)$ and $\psi(\omega)$ have the same laws under Q as γ and ω under P, since β and ν are deterministic. It remains to notice that (2.3) can be rewritten as

$$\begin{aligned} \frac{dQ}{dP} &= \exp\left(-\int_0^\infty z(s)d\omega(s) - \int_{\mathbb{R}_+\times[-1,1]^d} Z\circ\phi(s,y)\nu(ds,dy) - \frac{1}{2}\int_0^t z^2(s)d\beta(s)\right) \\ &\quad\times\prod_{(s,y)\in\gamma}(1+Z\circ\phi(s,y)), \end{aligned} \tag{2.4}$$

since

$$\Delta\tilde{M}(\tilde{s},\tilde{y}) = \begin{cases} 1 & \text{if } \exists\ (s,y)\in\gamma\ :\ (\tilde{s},\tilde{y}) = \phi(s,y), \\ 0 & \text{otherwise,}\end{cases}$$

and

$$\begin{aligned} \int_{\mathbb{R}_+\times[-1,1]^d} Z(s,y)\tilde{\nu}(ds,dy) &= \int_{\mathbb{R}_+\times[-1,1]^d}\frac{Z(s,y)}{1+Z(s,y)}\nu(ds,dy) \\ &= \int_{\mathbb{R}_+\times[-1,1]^d} Z\circ\phi(s,y)\nu(ds,dy). \end{aligned}$$

□

We make some remarks on different factorizations of the density function and on the deterministic case.

- If the process Z does not belong to $L^1(\mathbb{R}_+ \times [-1,1]^d, \tilde{\nu})$, P-a.s., then the product (2.4) may not converge. However, if Z is predictable and belongs to $L^2(\Omega) \otimes L^2(\mathbb{R}_+ \times [-1,1]^d, \tilde{\nu})$, i.e.

$$E_P\left[\int_{\mathbb{R}_+ \times [-1,1]^d} |Z \circ \phi(s,y)|^2 \nu(ds,dy)\right] = E_P\left[\int_{\mathbb{R}_+ \times [-1,1]^d} |Z(s,y)|^2 \tilde{\nu}(ds,dy)\right] < \infty,$$

then the probability density dQ/dP can still be formulated via a "renormalization" due to stochastic calculus. Namely, it can be written as the limit L_∞ as t goes to infinity of the process

$$\begin{aligned} L_t &= \exp\left(-\int_0^t z(s) d\tilde{M}^c(s) + \int_0^t \int_{[-1,1]^d} Z(s,y)(\tilde{\mu}(ds,dy) - \tilde{\nu}(ds,dy))\right. \\ &\quad \left. -\frac{1}{2}\int_0^t z^2(s) d\tilde{\beta}(s)\right) \prod_{(s,y),\ s\leq t} (1 + Z(s,y)\Delta\tilde{M}(s,y)) e^{-Z(s,y)\Delta\tilde{M}(s,y)}, \quad (2.5) \end{aligned}$$

$t \in \mathbb{R}_+$, which is solution to the Itô stochastic differential equation

$$\frac{1}{L_{t^-}} dL_t = z(t) d\tilde{M}^c(t) + \int_{[-1,1]^d} Z(t,y)(\tilde{\mu}(dt,dy) - \nu(dt,dy)), \quad L_0 = 1, \quad t > 0.$$

The factorization

$$\prod_{(s,y),\ s\leq t} (1 + Z(s,y)\Delta\tilde{M}(s,y)) e^{-Z(s,y)\Delta\tilde{M}(s,y)} \qquad (2.6)$$

is used in (2.5) because this modified product still converges as $t \longrightarrow \infty$ when $Z \in L^2(\Omega) \otimes L^2(\mathbb{R}_+ \times [-1,1]^d, \tilde{\nu})$ since $P(d\gamma)$ almost surely, the series

$$\sum_{(s,y)\in\gamma} (Z \circ \phi(s,y))^2$$

is summable. Moreover, in (2.5) the stochastic integral

$$\int_0^t \int_{[-1,1]^d} Z(s,y)(\tilde{\mu}(ds,dy) - \tilde{\nu}(ds,dy))$$

makes sense in $L^2(\Omega)$ for predictable $Z \in L^2(\Omega) \otimes L^2(\mathbb{R}_+ \times [-1,1]^d, \tilde{\nu})$. The same factorization will be used in the anticipating case, where the traditional determinant is replaced by a Carleman-Fredholm determinant (see (4.2)), compensated with a divergence operator, cf. [17], [23] (see below in Sect. 4).

- The absolute continuity result for deterministic shifts on Poisson space of [3], [18], [22] follows from the Girsanov theorem, in the particular case of a smooth deterministic diffeomorphism

$$\phi : \mathbb{R}_+ \times [-1,1]^d \longrightarrow \mathbb{R}_+ \times [-1,1]^d.$$

We denote by P_ν the Poisson measure on $\Gamma(\mathbb{R}_+ \times [-1,1]^d)$ with deterministic intensity ν, i.e. the probability under which $\mu(\omega,\gamma) = \gamma$ has intensity ν, with characteristic function

$$E_{P_\nu}\left[\exp\left(\int_{\mathbb{R}_+\times[-1,1]^d} g d\gamma\right)\right] = \exp\left(\int_{\mathbb{R}_+\times[-1,1]^d} (e^{g(y)}-1)\nu(dy)\right),$$

$g \in \mathcal{C}_c(\mathbb{R}_+ \times [-1,1]^d)$. Th. 2.1 gives

$$\begin{aligned} & E_P[f(\omega,\gamma)] \\ = \;& E_P\left[f(\omega,\phi(\gamma))\exp\left(-\int_{\mathbb{R}^d} Z\circ\phi(s,y)\nu(ds,dy)\right)\prod_{(s,y)\in\gamma}(1+Z\circ\phi(s,y))\right], \end{aligned}$$

$f : \Omega \longrightarrow \mathbb{R}$ measurable bounded, where $(1+Z)^{-1}$ is the density of $\phi_*\nu$ with respect to ν. In the particular case where ν is the Lebesgue measure on $\mathbb{R}_+ \times [-1,1]^d$ we find:

$$\frac{d\phi_*^{-1}P_\nu}{dP_\nu} = \exp\left(\int_{\mathbb{R}_+\times[-1,1]^d} (|\partial\phi(y)|-1)dy\right)\prod_{(s,y)\in\gamma}|\partial\phi(s,y)|.$$

3 Sequence model for the space Ω

In this section we introduce the sequence model of Ω as a vector space denoted by B, i.e. the random element (ω,γ) is constructed via a sequence of vectors which have independent Gaussian, exponential and uniformly distributed components. In the remaining of this paper we work in the case of a deterministic flat intensity ν given as

$$d\nu(y^1,\ldots,y^{d+1}) = \frac{1}{2^d}1_{\mathbb{R}_+}(y^1)1_{[-1,1]}(y^2)\cdots 1_{[-1,1]}(y^{d+1})dy^1\cdots dy^{d+1}, \qquad (3.1)$$

and with the quadratic variation $\beta(t) = t$, $t \in \mathbb{R}_+$. (We will use the notation $\mathbb{R}^{d+2} \ni x_k = (x_k^0,\ldots,x_k^{d+1})$). Let $B = \left\{x = (x_k)_{k\in\mathbb{N}} \;:\; x_k \in \mathbb{R}^{d+2}\right\}$, with the norm

$$\|x-y\|_B = \sup_{k\geq 0}\|x_k - y_k\|_{\mathbb{R}^{d+2}}/(k+1),$$

and associated Borel σ-algebra $\mathcal{F}$. Let $d \geq 1$, $r \geq 0$, and let λ_r be the finite measure on $\mathbb{R}^{d+2}$ defined by its density

$$\begin{aligned} & d\lambda_r(t_0, t_1, \ldots, t_{d+1}) \\ & = \frac{1}{2^{d-1}\sqrt{2\pi}} e^{-t_0^2/2} e^{-t_1} 1_{\mathbb{R}}(t_0) 1_{[-r,\infty[}(t_1) 1_{[-r-1,1+r]}(t_2) \cdots 1_{[-r-1,1+r]}(t_{d+1}) dt_0 \cdots dt_{d+1}. \end{aligned}$$

We denote by P the probability defined on $(B, \mathcal{F})$ via its expression on cylinder sets:

$$P(\{x = (x_k)_{k\in\mathbb{N}} \in B \ : \ (x_0, \ldots, x_n) \in A\}) = \lambda_0^{\otimes n+1}(A), \tag{3.2}$$

A Borel set in $(\mathbb{R}^{d+2})^{n+1}$, $n \in \mathbb{N}$. We denote by

$$\tau_k = (\tau_k^0, \ldots, \tau_k^{d+1}) : B \longrightarrow \mathbb{R}^{d+2} \quad k \in \mathbb{N},$$

the coordinate functionals defined as

$$\tau_k(x) = x_k = (x_k^0, \ldots, x_k^{d+1}),$$

and

$$(\tau_k^0(x), \ldots, \tau_k^{d+1}(x)) = (x_k^0, \ldots, x_k^{d+1}).$$

The sequences $(\tau_k^0)_{k\in\mathbb{N}}$, $(\tau_k^1)_{k\in\mathbb{N}}$, $(\tau_k^i)_{k\in\mathbb{N}}$, $i = 2, \ldots, d+2$, are independent and respectively Gaussian, exponential and uniform on $[-1, 1]$. We let

$$E = \mathbb{R}\times]0, \infty[\times] - 1, 1[^d,$$

$$\bar{E} = \mathbb{R} \times [0, \infty[\times[-1, 1]^d,$$

and

$$B_+ = \left\{x \in B \ : \ x_k \in \bar{E}, \ k \in \mathbb{N}\right\},$$

$$B_- = \{x \in B \ : \ x_k \in E, \ k \in \mathbb{N}\}.$$

The random configurations γ can be constructed as the sets of points

$$\gamma = \{T_k(x) \ \ : \ \ k \geq 1\} \subset \mathbb{R}_+ \times [-1, 1]^d, \quad x \in B_+,$$

defined as

$$T_k(x) = \left(\sum_{i=0}^{i=k-1} \tau_i^1(x), \tau_k^2(x), \ldots, \tau_k^{d+1}(x) \right), \quad x \in B, \quad k \geq 1.$$

On the other hand, it is well-known that the classical Brownian motion on $[0,1]$ can be constructed as

$$W(t)=t\tau_0^0+\sqrt{2}\sum_{n=1}^{\infty}\frac{\tau_n^0}{2n\pi}\sin(2n\pi t),\quad t\in[0,1],$$

i.e.

$$\tau_n^0=\sqrt{2}\int_0^1\sin(2\pi nt)dW(t),\ n\geq 1,\qquad \tau_0^0=\int_0^1 dW(t)=W(1),$$

and if $(z(t))_{t\in[0,1]}$ is an adapted process given as

$$z(t)=F(0,0)+\sqrt{2}\sum_{n=1}^{\infty}F(n,0)\cos(2n\pi t),\quad t\in[0,1],$$

then the stochastic integral of $(z(t))_{t\in[0,1]}$ with respect to $(W(t))_{t\in[0,1]}$ is written as

$$\int_0^1 z(t)dW(t)=\sum_{n=0}^{\infty}F(n,0)\tau_n^0,$$

and we have

$$\int_0^1 z^2(t)dt=\sum_{n=0}^{\infty}(F(n,0))^2.$$

Let also

$$E_i=\begin{cases}\mathbb{R}^{d+2}, & i=0,\\ \left\{(y^0,\ldots,y^{d+1})\in\mathbb{R}^{d+2}\ :\ y^1=0\right\}, & i=1,\\ \left\{(y^0,\ldots,y^{d+1})\in\mathbb{R}^{d+2}\ :\ y^i\in\{-1,1\}\right\}, & i=2,\ldots,d+1,\end{cases}$$

and

$$B_k^i=\{x\in B\ :\ x_k\in E_i\},\quad k\in\mathbb{N},\ i=1,\ldots,d+1.$$

We denote by $(e_k)_{k\geq 0}$ the canonical basis of $H=l^2(\mathbb{N},\mathbb{R}^{d+2})=l^2(\mathbb{N})\otimes\mathbb{R}^{d+2}$, with

$$e_k=(e_k^0,\ldots,e_k^{d+1}),\quad k\in\mathbb{N}.$$

In this framework, the shift of Brownian motion by a process $(\psi(s))_{s\in[0,1]}$ and the random diffeomorphism $\phi:\mathbb{R}_+\times[-1,1]^d\longrightarrow\mathbb{R}_+\times[-1,1]^d$ will be replaced by a random variable $F:B\longrightarrow H$ whose components are denoted by $(F(k,i))_{k\in\mathbb{N},i=0,\ldots,d+1}$.

The link between F and ψ, ϕ is the following:

$$F(k,0) = \begin{cases} \sqrt{2}\int_0^1 \sin(2\pi kt)\psi(t)dt, & k \geq 1, \\ \int_0^1 \psi(t)dt, & k = 0, \end{cases}$$

$$\tau_k^1 + F(k,1) = \phi^1(T_{k+1}) - \phi^1(T_k), \quad k \geq 0,$$

$$\tau_k^i + F(k,i) = \phi^i(T_k), \quad k \geq 0, \ i = 2, \dots, d+1.$$

4 Anticipating Girsanov theorem

In this section we will state the extension of the Girsanov theorem for Lévy processes to non-adapted shifts, and compare it to its classical adapted version (Th. 2.2). Before that we need to introduce the tools of gradient and divergence operator which will be used in the expressions of densities. Given X be a real separable Hilbert space with orthonormal basis $(h_i)_{i\in\mathbb{N}}$, let $H \otimes X$ denote the completed Hilbert-Schmidt tensor product of H with X. Let $\mathcal{S}$ be the set of functionals on B of the form $f(\tau_{k_1}, ..., \tau_{k_n})$, where $n \in \mathbb{N}$, $k_1, ..., k_n \in \mathbb{N}$, and f is a polynomial or $f \in \mathcal{C}_c^\infty(E^n)$. We define a set of smooth vector-valued functionals as

$$\mathcal{S}(X) = \left\{ \sum_{i=0}^{i=n} F_i h_i \ : \ F_0, \dots, F_n \in \mathcal{S}, \ h_0, \dots, h_n \in X, \ n \in \mathbb{N} \right\},$$

which is dense in $L^2(B, P; X)$.

Definition 4.1 *We define a gradient $D : \mathcal{S}(X) \to L^2(B, H \otimes X)$ by*

$$(DF(x), h)_{H\otimes X} = \lim_{\varepsilon\to 0} \frac{F(x+\varepsilon h) - F(x)}{\varepsilon}, \qquad x \in B, \ h \in H.$$

The coordinates of $DF \in L^2(B; H \otimes X)$ are denoted by $(D_k^i F)_{(k,i)\in\mathbb{N}\times\{0,\dots,d+1\}}$. For $u \in \mathcal{S}(H \otimes X)$, we write

$$u = \sum_{k=0}^{\infty} \sum_{i=0}^{i=d-1} u_k^i e_k^i, \quad u_k \in \mathcal{S}(X), \quad k \in \mathbb{N}.$$

Let

$$\mathcal{U}(X) = \left\{ u \in \mathcal{S}(H \otimes X) \ : \ u_k^i = 0 \text{ on } B_k^i, \ k \in \mathbb{N}, \ i = 0, \dots, d+1 \right\},$$

which is dense in $L^2(B; H \otimes X)$.

Proposition 4.1 *The operator $D : L^2(B;X) \to L^2(B; H \otimes X)$ is closable and has an adjoint operator $\delta : \mathcal{U}(X) \to L^2(B;X)$, with*

$$E_P\left[(DF,u)_{H\otimes X}\right] = E\left[(\delta(u),F)_X\right], \quad u \in \mathcal{U}(X), F \in \mathcal{S}(X), \tag{4.1}$$

where δ is defined as

$$\delta(u) = \sum_{k\in\mathbb{N}} \tau_k^0 u_k^0 + u_k^1 - \text{trace } D_k u_k, \quad u \in \mathcal{U}(X),$$

with

$$\text{trace } D_k u_k = D_k^0 u_k^0 + \cdots + D_k^{d+1} u_k^{d+1}, \quad u \in \mathcal{U}(X).$$

Proof. This result is proved by finite dimensional integration by parts with respect to λ_0, under the boundary conditions imposed on elements of $\mathcal{U}(X)$. □

Given a Hilbert-Schmidt operator $K : H \longrightarrow H$, the Carleman-Fredholm determinant of $I_H + K$ is defined as

$$\det{}_2(I_H + K) = \prod_{i=0}^{\infty}(1+\alpha_i)\exp(-\alpha_i), \tag{4.2}$$

where $(\alpha_k)_{k\in\mathbb{N}}$ are the eigenvalues of K, counted with their multiplicities, cf. [6].

Theorem 4.1 *Let $F : B \to H$ be such that $h \mapsto F(x+h)$ is continuously differentiable in $H \otimes H$ on $\{h \in H \ : \ x + h \in B_+\}$, a.s. for $x \in B$. Assume that*

(i) $(I_B + F)(B_-) = B_-$,

(ii) $(I_B + F)(B_k^i) \subset B_k^i$, $k \in \mathbb{N}$, $i = 1, \ldots, d+1$,

(iii) $I_B + F : B \longrightarrow B$ *is a.s. bijective,*

(iv) $I_H + DF : H \longrightarrow H$ *is a.s. invertible.*

Then

$$E_P[f] = E_P\left[|\det{}_2(I_H + DF)| \exp\left(-\delta(F) - \frac{1}{2}|\pi^0 F|_H^2\right) f \circ (I_B + F)\right]$$

for $f : B \longrightarrow \mathbb{R}$ measurable and bounded, where $\pi^0 : H \longrightarrow l^2(\mathbb{N})$ is the projection operator defined as $\pi^0(u) = (u_k^0)_{k\in\mathbb{N}}$.

This result is a particular case of Th. 4.2 stated below and proved in Sect. 6. The integrability condition (2.2) in Th. 2.1 is ensured by the hypothesis of Th. 4.1. We also make the following remarks:

- The boundary condition (ii) in Th. 4.1 is natural. For $i = 1$, it states that if two points in γ have same jump times then their images by $I_B + F$ also have same jump times. For $i = 2, \ldots, d+1$, it means that

$$\tau_k^i = \pm 1 \;\Rightarrow\; F(k,i) = 0$$

i.e. if a point lies at a boundary of $[-1,1]^d$, then its image by $I_B + F$ lies at the same boundary.

- Let us check that in the adapted Poisson case ($F(n,0) = 0$, $n \in \mathbb{N}$), the above result is in agreement with Th. 2.1 (ν is the flat intensity given by (3.1)). If $I_B + F : B \longrightarrow B$ satisfies the hypothesis of Th. 4.1 and corresponds to a smooth $\mathcal{F}_t$-predictable random diffeomorphism $\phi : \mathbb{R}_+ \times [-1,1]^d \longrightarrow \mathbb{R}_+ \times [-1,1]^d$ satisfying the hypothesis of Th. 2.1, then $D_k^i F(l,j) = 0$, $k > l$, hence $I_H + DF$ is a block diagonal matrix, each $d \times d$ diagonal block being equal to the Jacobian determinant $|\partial\phi(T_k)|$. We have $\tau_k^1 + F(k,1) = \phi^1(T_{k+1}) - \phi^1(T_k)$, $k \geq 1$, $\phi^i(T_k) = \pm 1$ if $\tau_k^i = \pm 1$, $k \geq 1$, $i = 2, \ldots d+1$, and

$$\begin{aligned}
\sum_{k=0}^{\infty} F(k,1) &= \lim_{k\to\infty} (\phi^1(T_k) - T_k^1) \\
&= \lim_{k\to\infty} \nu(\phi([0,T_k] \times [-1,1]^d)) - \nu([0,T_k] \times [-1,1]^d) \\
&= \int_{\mathbb{R}_+ \times [-1,1]^d} (|\partial\phi(s,x)| - 1)\nu(ds,dx), \quad a.s.
\end{aligned}$$

Hence the formula of Th. 2.1:

$$\begin{aligned}
&|\det{}_2(I_H + DF)| \exp\left(-\delta(F)\right) = |\det(I_H + DF)| \exp\left(-\sum_{k=0}^{\infty} F(k,1)\right) \\
&= \exp\left(-\sum_{k=0}^{\infty} F(k,1)\right) \prod_{k=1}^{\infty} |\partial\phi(T_k)| \\
&= \exp\left(-\int_{\mathbb{R}_+ \times [-1,1]^d} (|\partial\phi(s,x)| - 1)\nu(ds,dx)\right) \prod_{k=1}^{\infty} |\partial\phi(T_k)|.
\end{aligned}$$

- Still in the adapted Poisson case, the Carleman-Fredholm factorization $\det_2(I_H + DF)$ of the determinant has some similarity with the expression (2.5) of the density $L_\infty = dQ/dP$. The conditions $Z = (|\partial\phi| - 1) \in L^1(\mathbb{R}_+ \times [-1,1]^d, \tilde{\nu})$, resp. $Z(|\partial\phi| - 1) \in L^2(\mathbb{R}_+ \times [-1,1]^d, \tilde{\nu})$ are the respective analog of the trace class and Hilbert-Schmidt hypothesis on DF, a.s. More precisely we have

$$\det(I_H + DF) = \prod_{k=1}^{\infty} |\partial\phi(T_k)| \exp\left(-\sum_{i=1}^{d+1} D_k^i F(k,i)\right).$$

  If (and only if) $d = 0$, then the Carleman-Fredholm determinant $\det_2(I_H + DF)$ coincides exactly with the factorization (2.6), i.e.

$$\det_2(I_H + DF) = \prod_{k=1}^{\infty} |\phi'(T_k)| \exp\left(1 - \phi'(T_k)\right),$$

  and $\delta(F) = \int_0^\infty \phi'(t)(\gamma(dt) - \nu(dt))$.

- In the general adapted case, i.e. if F also perturbs the Brownian component, then this type of result can not be checked via the above elementary computation. If $I_B + F$ satisfies all the above smoothness and adaptedness conditions, the comparison of Th. 4.1 to the classical Girsanov theorem Th. 2.1 yields an equality between

$$\begin{aligned}
&|\det_2(I_H + DF)| \exp\left(-\delta(F) - \frac{1}{2}|\pi^0 F|_H^2\right) \\
&= |\det(I_H + DF)| \exp\left(-\sum_{k=0}^{\infty} \tau_k^0 F(k,0) - F(k,1) - \frac{1}{2}(F(k,0))^2\right) \\
&= \exp\left(-\int_0^1 z(s)dW(s) + \int_{\mathbb{R}_+\times[-1,1]^d} (|\partial\phi(s,x)| - 1)\nu(ds,dx) - \frac{1}{2}\int_0^1 z(s)^2 ds\right) \\
&\quad \times |\det(I_H + DF)|
\end{aligned}$$

  and

$$\exp\left(-\int_0^1 z(s)dW(s) + \int_{\mathbb{R}_+\times[-1,1]^d} (|\partial\phi(s,x)| - 1)\nu(ds,dx) - \frac{1}{2}\int_0^1 z(s)^2 ds\right) \times \prod_{k=1}^{\infty} |\partial\phi(T_k)|,$$

  hence the expression of the determinant in the case of an adapted transformation of a Brownian motion and a Poisson random measure satisfying the hypothesis of Th. 2.1 and Th. 4.1:

$$\det(I_H + DF) = \prod_{k=1}^{\infty} |\partial\phi(T_k)|.$$

We will prove a result which is more general than Th. 4.1 and does not require $I_B + F$ to be bijective. For this we need to consider the following class of transformations, cf. [10], [21].

Definition 4.2 *A random variable $F : B \to H$ is said to be $\mathcal{HC}^1_{loc}$ if there is a random variable $R : B \longrightarrow [0, \infty]$ with $R > 0$ a.s. such that $h \to F(x + h)$ is continuously differentiable in $H \otimes H$ on*

$$\{h \in H \ : \ \|h\|_H < R(x) \text{ and } x + h \in B_+\},$$

for any $x \in B_+$.

Our main result is the following, it is formulated for not necessarily invertible shifts, as in [21] on the Wiener space.

Theorem 4.2 *Let $F \in \mathcal{HC}^1_{loc}$ and $M = \{x \in B_+ \ : \det_2(I_H + DF) \neq 0\}$. Assume that*

(i) $(I_B + F)(B_-) \subset B_-$ and

(ii) $(I_B + F)(B^i_k) \subset B^i_k$, $k \in \mathbb{N}$, $i = 1, \ldots, d + 1$.

Then

(i) $N(x; M) := card((I_B + F)^{-1}(x) \bigcap M)$, $x \in B$, is at most countably infinite,

(ii) we have

$$E_P[fN(\cdot; M)] = E_P\left[|\det_2(I_H + DF)| \exp\left(-\delta(F) - \frac{1}{2}|\pi^0 F|^2_H\right) f \circ (I_B + F)\right]$$

for $f \in \mathcal{C}^+_b(B)$,

(iii) the measure $(I_B + F)_*(P_{|M})$ is absolutely continuous with respect to P, and

$$\frac{d(I_B + F)_* P_{|M}}{dP}(x) = \sum_{\theta \in (I_B + F)^{-1}(x) \cap M} \exp\left(\delta(F)(\theta) + \frac{1}{2}|\pi^0 F(\theta)|^2_H\right) \frac{1}{|\det_2(I_H + DF(\theta))|}.$$

Proof. cf. Sect. 6. □

Th. 4.1 is a particular case of Th. 4.2 with $R = \infty$, P-a.s., $P(M) = 1$ and $I_B + F$ P-a.s. bijective.

5 Technical results

In this section, some further notation is introduced, and basic properties of D and δ are stated.

Definition 5.1 *For $p \geq 1$, we call*

- $\mathbb{D}_{p,1}(X)$ *the completion of $\mathcal{S}(X)$ with respect to the norm*

$$\|F\|_{\mathbb{D}_{p,1}(X)} = \| \|F\|_X \|_{L^p(B)} + \| \|DF\|_{H\otimes X} \|_{L^p(B)},$$

- $\mathbb{D}^{\mathcal{U}}_{p,1}(H)$ *the completion of $\mathcal{U}(\mathbb{R})$ with respect to the norm $\|\cdot\|_{\mathbb{D}_{p,1}(H)}$,*
- $\mathbb{D}_{\infty,1}(X)$*, resp.* $\mathbb{D}^{\mathcal{U}}_{\infty,1}(H)$ *the subset of* $\mathbb{D}_{2,1}(X)$*, resp.* $\mathbb{D}^{\mathcal{U}}_{2,1}(H)$ *made of the random variables F for which $\|F\|_{\mathbb{D}_{\infty,1}(X)}$, resp. $\|F\|_{\mathbb{D}_{\infty,1}(H)}$ is bounded.*

For $p \in [1,\infty]$, we call

- $\mathbb{D}^{loc}_{p,1}(X)$*, resp.* $\mathbb{D}^{\mathcal{U},loc}_{p,1}(H)$*, the sets of functionals F such that there is a measurable almost sure partition $(A_n)_{n\in\mathbb{N}}$ of B and $F_n \in \mathbb{D}_{p,1}(X)$, resp. $F_n \in \mathbb{D}^{\mathcal{U}}_{p,1}(X)$, with $F_n = F$ a.s. on A_n, $n \in \mathbb{N}$.*

For $p = 2$,

- *let* $\mathrm{Dom}(\delta; X)$ *denote the domain of the closed extension of δ.*

The interest in the space $\mathbb{D}^{\mathcal{U}}_{2,1}(H)$ is that it is a Hilbert space contained in $\mathrm{Dom}(\delta;X)$, as shown in the following proposition.

Proposition 5.1 *(i) The operator δ is continuous from $\mathbb{D}^{\mathcal{U}}_{2,1}(H)$ into $L^2(B)$ with*

$$\|\delta(F)\|^2_{L^2(B)} \leq (d+2)\|F\|^2_{\mathbb{D}^{\mathcal{U}}_{2,1}(H)}, \quad F \in \mathbb{D}^{\mathcal{U}}_{2,1}(H). \tag{5.1}$$

(ii) The operators D and δ are local, cf. [2], [12], [14], i.e. for $F \in \mathbb{D}_{2,1}(X)$, resp. $F \in \mathrm{Dom}(\delta;X)$. we have $DF = 0$ a.s. on $\{F = 0\}$, resp. $\delta(F) = 0$ a.s. on $\{F = 0\}$.

Proof. (i) Let $F \in \mathcal{U}(\mathbb{R})$. We have

$$\delta(F) = \sum_{k=0}^{\infty} \left(\tau^0_k F(k,0) + F(k,1) - \sum_{i=0}^{d+1} D^i_k F(k,i) \right),$$

and

$$
\begin{aligned}
(\delta(F))^2 \;\leq\; & (d+2)\left(\sum_{k=0}^{\infty}\tau_k^0 F(k,0) - D_k^0 F(k,0)\right)^2 \\
& +(d+2)\left(\sum_{k=0}^{\infty} F(k,1) - D_k^1 F(k,1)\right)^2 + (d+2)\sum_{i=2}^{d+1}\left(\sum_{k=0}^{\infty} D_k^i F(k,i)\right)^2,
\end{aligned}
$$

hence from the Gaussian, exponential and uniform cases, cf. [17], [14], [15], we have

$$
\begin{aligned}
\|\delta(F)\|^2_{L^2(B)} \;\leq\; & (d+2)E_P\left[\sum_{k=0}^{\infty}(F(k,0))^2\right] \\
& +(d+2)E_P\left[\sum_{k,l=0}^{\infty}(D_k^0 F(l,0))^2 + (D_k^1 F(l,1))^2 + \sum_{i=2}^{d+1}(D_k^i F(l,i))^2\right] \\
\leq\; & (d+2)\|\pi^0 F\|^2_{I\!D^{\mathcal{U}}_{2,1}(H)}.
\end{aligned}
$$

(See Th.4.1 for the definition of π^0). The proof of (ii) relies only on the duality relation between D and δ and on the density of $\mathcal{U}(X)$ in $L^2(B; H \otimes X)$. □

The proof of the following result is directly adapted from [4], [11], [13], [14], it stays valid by replacing λ_0 with any absolutely continuous probability measure on $\mathbb{R}^{d+2}$. Let $\mathcal{F}_n$ denote the σ-algebra generated by $\tau_0, \ldots, \tau_n$.

Lemma 5.1 *Let $F \in L^2(B; X)$ and $F_n = E[F \mid \mathcal{F}_n] \in I\!D_{2,1}(X)$, $n \in \mathbb{N}$.*

- *$F \in I\!D_{2,1}(X)$ if and only if $F_n \in I\!D_{2,1}(X)$ $\forall n \in \mathbb{N}$ and $(DF_n)_{n\in\mathbb{N}}$ converges in $L^2(B; H \otimes X)$. In this case,*

$$\|DF_n\|_{H\otimes X} \leq \|DF\|_{H\otimes X}, \quad a.s., \; n \in \mathbb{N}.$$

- *F_n belongs to $I\!D_{2,1}(\mathbb{R})$ if and only if there exists*

$$f \in W^{2,1}(E^{n+1}, \lambda_0^{\otimes n+1})$$

such that $F_n = f(\tau_0, \ldots, \tau_n)$. In this case, $DF_n = (\partial_k f(\tau_0, \ldots, \tau_n))_{k\in\mathbb{N}}$.

- *Assume that for some $c > 0$,*

$$\|F(x+h) - F(x)\|_X \leq c\|h\|_H, \quad h \in H, \; x \in B_+, \; x+h \in B_+.$$

Then $F \in I\!D_{2,1}(X)$ and $\|DF\|_{H\otimes X} \leq c$, a.s.

We denote by π_n the application $\pi_n : B \longrightarrow H$ defined by $\pi_n(x) = \left(x_k 1_{\{k\leq n\}}\right)_{k\in\mathbb{N}}$.

Lemma 5.2 *Let $F : B \to H$ measurable and bounded such that*

(i) $(I_B + F)(B_k^i) \subset B_k^i$, $k \in \mathbb{N}$, $i = 1, \dots, d+1$,

(ii) F is Lipschitz on B_+ with Lipschitz constant $c > 0$:

$$\|F(x+h) - F(x)\|_H < c\|h\|_H, \quad h \in H,\ x \in B_+,\ x+h \in B_+.$$

Then $F \in \mathbb{D}^{\mathcal{U}}_{\infty,1}(H)$, and there is a sequence $(F_n)_{n\in\mathbb{N}} \subset \mathcal{U}(\mathbb{R})$ that converges in $\mathbb{D}^{\mathcal{U}}_{2,1}(H)$ to F with

(i) $\|\|F_n\|_H\|_\infty \leq \|\|F\|_H\|_\infty$.

(ii) $\|\|DF_n\|_{H\otimes H}\|_\infty \leq c$, $n \in \mathbb{N}$.

Proof. Let

$$\tilde{E}_i = \begin{cases} E, & i = 0, \\ \mathbb{R} \times \mathbb{R} \times [-1,1]^d, & i = 1, \\ \mathbb{R} \times \mathbb{R}_+ \times [-1,1]^{i-2} \times \mathbb{R} \times [-1,1]^{d-i+1}, & i = 2, \dots, d+1, \end{cases}$$

and

$$\tilde{B}_k^i = \{x \in B_+ \ : \ x_k \in \tilde{E}_i\}, \quad i = 0, \dots, d+1,\ k \in \mathbb{N}.$$

First we note that after putting $F = 0$ on B_+^c, the Lipschitz condition on F extends to $\tilde{B}_k^i$ as

$$|F(k,i)(x+h) - F(k,i)(x)| < c\|h\|_H, \quad h \in H,\ x \in \tilde{B}_k^i,\ x+h \in \tilde{B}_k^i.$$

Let $F_n = \pi_n E[F \mid \mathcal{F}_n]$, $n \in \mathbb{N}$. The sequence $(F_n)_{n\in\mathbb{N}}$ converges to F in $\mathbb{D}_{2,1}(H)$ and satisfies to (i) and (ii). There exists a function $f_k^i \in W^{2,1}(\mathbb{R}^{(d+2)(n+1)}, dx)$ which has a Lipschitz version on $\mathbb{R}^{(d+2)(n+1)}$ and support in $\bar{E}^{n+1}$, such that $F_n(k,i) = f_k^i(\tau_0, \dots, \tau_n)$ P-a.e., $k = 0, \dots, n$, $i = 0, \dots, d+1$. Let $\Psi \in \mathcal{C}_c^\infty(\mathbb{R}^{(d+2)(n+1)})$ with support in $[-2,0]^{k(d+2)+i} \times [0,2] \times [-2,0]^{(n-k)(d+2)+(d+2-i)}$, $0 \leq \Psi \leq 1$ and $\int_{\mathbb{R}^{(d+2)(n+1)}} \Psi(x)dx = 1$. Let for $N \geq 2$ and $y \in \bar{E}^{n+1}$:

$$\phi_{k,i,N}(y) = \begin{cases} \left(\frac{1}{N}\right)^{(d+2)(n+1)} \int_{\tilde{E}_i} \Psi(N(y-x)) f_k^i(x)dx, \ i = 0, 1, \\ \left(\frac{1}{N}\right)^{(d+2)(n+1)} \int_{\tilde{E}_i} \Psi(N(y-x)) f_k^i(x)dx, \ y_k^i < 0, \ i = 2, \dots, d+1, \\ \left(\frac{1}{N}\right)^{(d+2)(n+1)} \int_{\tilde{E}_i} \Psi(N(y+x)) f_k^i(x)dx, \ y_k^i \geq 0, \ i = 2, \dots, d+1. \end{cases}$$

For $k \in \mathbb{N}$ and $i = 0, \ldots, d+1$, let $G_N(k,i) = \phi_{k,i,N}(\tau_0, \ldots, \tau_n)$, $k = 0, \ldots, n$, and $G_N(k,i) = 0$, $k > n$. Then $G_N \in \mathcal{U}(\mathbb{R})$, $N \geq 2$, and $(G_N)_{N\geq 2}$ converges to F_n in $\mathbb{D}_{2,1}(H)$ and satisfies to (i) and (ii). $\square$

We refer to [10] for the following definition on the Wiener space.

Definition 5.2 *If $A \subset B$ is measurable, let*

$$\rho_A(x) = \begin{cases} \inf_{h\in H} \{\|h\|_H \ : \ x+h \in A\}, & x \in B, \\ \infty, & x \notin A+H, \ x \in B. \end{cases}$$

Let $\phi \in \mathcal{C}_c^\infty(\mathbb{R})$ with $\|\phi\|_\infty \leq 1$, such that $\phi = 0$ on $[2/3, \infty[$, $\phi = 1$ on $[0, 1/3]$ and $\|\phi'\|_\infty < 4$. If A σ-compact, then

$$\|\phi(\rho_A(x+h)) - \phi(\rho_A(x))\|_H \leq \|\phi'\|_\infty \|h\|_H, \quad x \in B, \ h \in H,$$

hence $\phi(\rho_A) \in \mathbb{D}_{\infty,1}(\mathbb{R})$ with $\|D\phi(\rho_A)\|_H \leq \|\phi'\|_\infty$.

Lemma 5.3 *Let $n_0 \in \mathbb{N}$ and $F \in \mathcal{HC}^1_{loc}$, such that*

$$(I_B + F)(B_k^i) \subset B_k^i, \quad i = 1, \ldots, d+1, \quad k > n_0.$$

Let $\alpha, \beta > 0$ and

$$A = \left\{ x \in B_- \ : \ \begin{array}{l} \rho_{B_k^i}(\pi_{n_0} x) > 4/\alpha, \ k \in \mathbb{N}, \ i = 1, \ldots, d+1, \\ R(x) \geq 4/\alpha, \\ \sup\limits_{\|h\|_H \leq 2/\alpha} \|F(x+h)\|_H \leq \beta/(6\alpha), \\ \sup\limits_{\|h\|_H \leq 2/\alpha} \|DF(x+h)\|_{H\otimes H} \leq \beta/6 \end{array} \right\}$$

and $\tilde{F} = \phi(\alpha\rho_G)F$, where G is a σ-compact set contained in A. Then

$$\|\tilde{F}(x+h) - \tilde{F}(x)\|_H \leq (5\beta/6)\|h\|_H, \quad h \in H, \ x, x+h \in B_+,$$

and $\|\|\tilde{F}\|_H\|_\infty \leq \beta/(6a)$. Consequently $\tilde{F} \in \mathbb{D}^{\mathcal{U}}_{\infty,1}(H)$, and $F \in \mathbb{D}^{\mathcal{U},loc}_{\infty,1}(H)$.

Proof. Any $x \in A$ satisfies $x_k^1 > 4/\alpha$, $1 - x_k^i > 4/\alpha$, and $x_k^i + 1 > 4/\alpha$, $i = 2, \ldots, d+1$, $k \leq n_0$. For x in B_k^i we have $\rho_A(x) \geq 4/\alpha$, hence $\phi(\alpha\rho_A(x)) = 0$, and $(I_B + \tilde{F}(k,i))(B_k^i) \subset B_k^i$, $0 \leq k \leq n_0$, $i = 1, \ldots, d+1$. From Lemma 5.2 it follows that $\tilde{F} \in \mathbb{D}^{\mathcal{U}}_{\infty,1}(H)$. The fact that $F \in \mathbb{D}^{\mathcal{U},loc}_{\infty,1}(H)$ is proved by covering B with a countable collection of sets such as A, with $\alpha, \beta \in \mathbb{Q}\cap]0, \infty[$. $\square$

Lemma 5.4 *Let $F, G \in \mathcal{S}(H)$ and $T = I_B + F$. We have $G \circ T \in \mathrm{Dom}(\delta)$ and*

$$\delta(G) \circ T = \delta(G \circ T) + \mathrm{trace}\ (DF^*(DG) \circ T) + (\pi^0 F, G \circ T)_H.$$

Proof. We have $\delta(G \circ T) \in \mathcal{S}$ and

$$\begin{aligned}
\delta(G \circ T) &= \sum_{k=0}^{\infty} \tau_k^0 G(k,0) \circ T + G(k,1) \circ T - \sum_{k=0}^{\infty} \sum_{i=0}^{d+1} D_k^i \left(G(k,i) \circ T\right) \\
&= \sum_{k=0}^{\infty} \tau_k^0 G(k,0) \circ T + G(k,1) \circ T - \sum_{k=0}^{\infty} \sum_{i=0}^{i=d+1} D_k^i (I_B + F)^*(DG(k,i)) \circ T \\
&= -(\pi^0 F, G \circ T)_H + \delta(G) \circ T - \sum_{i,j=0}^{i=d+1} \sum_{k,l=0}^{\infty} D_k^i F(l,j)(D_l^j G(k,i)) \circ T \\
&= \delta(G) \circ T - \mathrm{trace}\ (DF^*(DG) \circ T) - (\pi^0 F, G \circ T)_H.
\end{aligned}$$

□

6 Proof of Th. 4.2

Since from Prop. 5.1, $I\!D_{2,1}^{\mathcal{U}}(H) \subset \mathrm{Dom}(\delta)$, we define

$$\Lambda_F = \det{}_2(I_H + DF) \exp\left(-\delta(F) - \frac{1}{2}|\pi^0 F|_H^2\right), \quad F \in I\!D_{2,1}^{\mathcal{U},loc}(H). \tag{6.1}$$

The proof of Th. 4.2 is done in two main steps: first we treat the case of Lipschitz transformations in the following lemma. Then we use the fact that $F \in \mathcal{HC}_{loc}^1$ can be locally written as a Lipschitz transformation, as in [10]. Let $P_n^\perp$ denote the image measure of P by $\pi_n^\perp := I_B - \pi_n$.

Proposition 6.1 *Let $n_0 \in \mathbb{N}$, let $K : H \to \pi_{n_0} H$ be a linear operator such that $I_H + K$ is invertible, and let $v \in \pi_{n_0} H$. Let $F : B \to H$ be measurable, bounded with bounded support, such that*

(i) $(I_B + F)(B_k^i) \subset B_k^i$, $k \in \mathbb{N}$, $i = 1, \ldots, d+1$,

(ii) $\pi_{n_0}^\perp \circ (I_B + F)(B) \subset B_+$, a.s.,

(iii) F is Lipschitz on B_- with Lipschitz constant $c < \left(\|(I_H + K)^{-1}\|_\infty\right)^{-1}$:

$$\|F(x+h) - F(x)\|_H \le c\|h\|_H, \quad h \in H,\ x \in B_-,\ x + h \in B_-. \tag{6.2}$$

Then $I_B + F + K + v$ is injective and there is $r > 0$ such that

$$E_P[f] = \int_B |\Lambda_{F+K+v}| \; f \circ (I_B + F + K + v) \; d\lambda_r^{\otimes(n_0+1)} \otimes P_{n_0}^{\perp},$$

for $f : B \longrightarrow \mathbb{R}$ bounded measurable with support in B_-.

Proof. *Step 1:* finite dimensional case. The injectivity of $I_B + F + K + v$ follows from (6.2) as in [10], [14]. Let $F = 0$ on B_+^c. Let $(F_n)_{n \geq n_0} \subset \mathcal{U}(\mathbb{R})$ be the sequence given by Lemma 5.2, converging to F in $\mathbb{D}_{2,1}(H)$, and let $T_n = I_B + F_n + K + v$. Since $\pi_{n_0}^{\perp}(I_B + F)(B) \subset B_+$, by construction the sequence $(F_n)_{n \in \mathbb{N}}$ also satisfies $\pi_{n_0}^{\perp}(I_B + F_n)(B) \subset B_+$, $n \in \mathbb{N}$. Replicating the argument of [10], [14], [21], we show that $I_B + F_n \circ (I_B + K)^{-1} + v$ is contractive from (6.2), hence bijective on B with inverse $I_B + G_n = (I_B + K) \circ T_n^{-1}$, where G_n satisfies

$$G_n = -F_n \circ (I_B + K)^{-1} \circ (I_B + G_n) - v, \tag{6.3}$$

and

$$\|DG_n\|_{H \otimes H} \leq c \|(I_H + K)^{-1}\|_{\infty} / (1 - c\|(I_H + K)^{-1}\|_{\infty}). \tag{6.4}$$

From (6.3) and the uniform boundedness in n and x of $(F_n)_{n \geq n_0}$ and $(G_n)_{n \geq n_0}$, there exists $r > 0$ such that $|T_n^{-1}(k, i)| < 1 + r$, $i = 2, \ldots, d+1$, and $T_n^{-1}(k, 1) > -r$, $n \in \mathbb{N}$, $k = 0, \ldots, n_0$, on B_+. Let $g \in \mathcal{C}^{\infty}(\mathbb{R}^{(d+2)(n+1)}, \pi_{n+1}H)$ such that $F_n + K + v = g(\tau_0, \ldots, \tau_n)$, $n \geq n_0$. Since $\pi_{n_0}^{\perp}(I_B + F_n)(B) \subset B_+$ and $F_n = 0$ on B_-, $n \in \mathbb{N}$, from (6.3) we have

$$T_n(\{x \in B \; : \; x_k \in \bar{E}, \quad k > n_0\}) = \{x \in B \; : \; x_k \in \bar{E}, \quad k > n_0\}, \tag{6.5}$$

hence $x \mapsto x + g(x)$ is a diffeomorphism of $(\mathbb{R}^{d+2})^{n_0} \times \bar{E}^{n-n_0}$. The Jacobi theorem in dimension $(d+2)(n+1)$ gives:

$$\begin{aligned}
&\int_B |\Lambda_{F_n+K+v}| \; f(\pi_n \circ T_n) \; d\lambda_r^{\otimes(n_0+1)} \otimes P_{n_0}^{\perp} \\
&= \frac{1}{2^{(d-1)(n+1)}} \int_{\mathbb{R}^{(d+2)(n+1)}} 1_{E^{n+1}}(x + g(x)) f(x + g(x)) \\
&\qquad |\det(I_{\mathbb{R}^{n+1}} + \partial g)| \exp\left(-\sum_{k=1}^{k=n} g_k^i + x_k^i + x_k^i g_k^0 + \frac{1}{2}(x_k^0)^2 + \frac{1}{2}(g_k^0)^2 \right) m(dx) \\
&= \frac{1}{2^{(d-1)(n+1)}} \int_{\mathbb{R}^{(d+2)(n+1)}} 1_{E^{n+1}}(y) f(y) \exp\left(-\sum_{k=1}^{k=n} y_k^1 + \frac{1}{2}(y_k^0)^2 \right) m(dy) \\
&= E_P[f \circ \pi_n], \qquad (6.6)
\end{aligned}$$

where $m(dx)$ denotes the Lebesgue measure on $\mathbb{R}^{(d+2)(n+1)}$, for $f \in \mathcal{C}_b^+(\mathbb{R}^{(d+2)(n+1)})$ with support in E^{n+1}. This relation extends to $f \in \mathcal{C}_b^+(B)$ with support in B_-:

$$\int_B |\Lambda_{F_n+K+v}| \; f \circ T_n \; d\lambda_r^{\otimes(n_0+1)} \otimes P_{n_0}^\perp = E_P\left[f\right].$$

Step 2: uniform integrability argument. From the de la Vallée-Poussin Lemma we need to find a bound on

$$\int_B 1_{B_-} \circ T_n |\Lambda_{F_n+K+v} \log |\Lambda_{F_n+K+v}|| \; d\lambda_r^{\otimes(n_0+1)} \otimes P_{n_0}^\perp,$$

uniformly on $n > n_0$. Since $(\|DF_n(x)\|_{H\otimes H})_{n\in\mathbb{N}}$ is bounded uniformly in $n \in \mathbb{N}$ and $x \in B$, $(|\det_2 DT_n(x)|)_{n\in\mathbb{N}}$ is uniformly lower and upper bounded from

$$|\det_2(DT_n(x))| \leq (1 + \|DT_n(x) - I_H\|_{H\otimes H}) \exp(1 + \|DT_n(x) - I_H\|_{H\otimes H}^2),$$

we only need to estimate

$$\begin{aligned}
&\int_B 1_{B_-} \circ T_n |\delta(F_n + K + v)\Lambda_{F_n+K+v}| \; d\lambda_r^{\otimes(n_0+1)} \otimes P_{n_0}^\perp \\
&= E_P\left[|\delta(F_n + K + v) \circ T_n^{-1}|\right] \\
&\leq E_P\left[|\delta(\pi_{n_0} F_n + K + v) \circ T_n^{-1}|\right] && (6.7)\\
&\quad + E_P\left[|\mathrm{trace}\left[\left(D\left(-K \circ (I_B + K)^{-1} + (I_B + K)^{-1} \circ G_n\right)\right)^* \cdot \left(D\pi_{n_0}^\perp F_n\right) \circ T_n^{-1}\right]|\right] \\
&\quad + E_P\left[|(\pi^0(G + G \circ (I_B + K)), \pi_{n_0}^\perp F_n \circ T_n^{-1})_H|\right] \\
&\quad + E_P\left[|\delta(\pi_{n_0}^\perp F_n \circ T_n^{-1})|\right], && (6.8)
\end{aligned}$$

from (6.6) and Lemma 5.4, since

$$\begin{aligned}
T_n^{-1} &= (I_B + K)^{-1}(I_B + G_n) \\
&= (I_B + K)^{-1} + (I_B + K)^{-1}(I_B + G_n) \\
&= (I_B + K)(I_B + K)^{-1} - K(I_B + K)^{-1} + (I_B + K)^{-1}(I_B + G_n) \\
&= I_B - K(I_B + K)^{-1} + (I_B + K)^{-1}(I_B + G_n), \quad n \in \mathbb{N}.
\end{aligned}$$

The first three terms in (6.8) are uniformly bounded in n from (6.4). From (6.3), we have $\pi_{n_0}^\perp G_n = -\pi_{n_0}^\perp F_n \circ T_n^{-1}$, and $\pi_{n_0}^\perp G_n \in \mathcal{U}(\mathbb{R})$, hence from (5.1),

$$\begin{aligned}
E_P\left[|\delta(\pi_{n_0}^\perp F_n \circ T_n^{-1})|\right] &= E_P\left[|\delta(\pi_{n_0}^\perp G_n)|\right] \leq E_P\left[|\delta(\pi_{n_0}^\perp G_n)|^2\right] \\
&\leq (d+2)E_P\left[\|D\pi_{n_0}^\perp G_n\|_{H\otimes H}^2\right] + (d+2)\|F\|_\infty^2 \\
&\leq (d+2)(c\|(I_H + K)^{-1}\|_\infty/(1 - c\|(I_H + K)^{-1}\|_\infty))^2 \\
&\quad + (d+2)\|F\|_\infty^2, \quad n \geq n_0,
\end{aligned}$$

from (5.1). Choosing a subsequence we have the $\lambda_r^{\otimes(n_0+1)} \otimes P_{n_0}^{\perp}$-a.e. convergence of $(|\Lambda_{F_n+K+v}| f \circ T_n)_{n \geq n_0}$ to $|\Lambda_{F+K+v}|\ f \circ T$, and by uniform integrability

$$\int_B |\Lambda_{F+K+v}|\ f \circ T\ d\lambda_r^{\otimes(n_0+1)} \otimes P_{n_0}^{\perp} = E_P[f]. \tag{6.9}$$

□

Proof. of Th. 4.2. We construct a family of sets that form a partition of M, such that F is Lipschitz and satisfies the hypothesis of Prop. 6.1 on each of those sets. Given $K : H \to \pi_{n_0} H$ a linear operator, $v \in \pi_{n_0} H$, $n > 8$, let

$$\begin{aligned} A(n_0, n, K, v) \quad = \quad & \Big\{\, x \in B_- : \rho_{B_k^i}(\pi_{n_0} x) > \frac{8}{n}, \quad k \in \mathbb{N},\ i = 1, \ldots, d+1, \\ & R(x) > \frac{4}{n}, \\ & \sup_{\|h\|_H \leq 1/n} \|F(x+h) - K(x+h) - v\|_H < \frac{1}{6n} \left(\|(I_H + K)^{-1}\|_\infty\right)^{-1}, \\ & \sup_{\|h\|_H \leq 1/n} \|DF(x+h) - K\|_{H \otimes H} < \frac{1}{n} \left(\|(I_H + K)^{-1}\|_\infty\right)^{-1} \Big\}, \end{aligned}$$

Let $F_{K,v} = \phi(n\rho_{\tilde{A}(n_0,n,K,v)})(F - K - v)$, where $\tilde{A}(n_0, n, K, v)$ is a σ-compact modification of $A(n_0, n, K, v) \bigcap M$. Then from Lemma 5.3, $F_{K,v}$ and $\tilde{A}(n_0, n, K, v)$ satisfy the hypothesis of Prop. 6.1, and since $T = I_B + F_{K,v} + K + v$ on $\tilde{A}(n_0, n, K, v) \subset B_-$, we have from Prop. 6.1:

$$E_P\left[1_{\tilde{A}(n_0,n,K,v)}\ |\Lambda_F|\ f \circ T\right] = E_P\left[1_{T(\tilde{A}(n_0,n,K,v))} f\right].$$

Finally we deal with the non-invertibility of $T = I_B + F$ as in [21]. Denote by $(\tilde{A}_k)_{k \in \mathbb{N}}$ the countable family $(\tilde{A}(n_0, n, K, v))_{n_0,n,K,v}$ obtained by letting K, resp. v, run in the finite rank linear operators and vectors with rational coefficients. Let $M_n = \tilde{A}_n \bigcap \left(\bigcup_{i=0}^{i=n-1} \tilde{A}_i\right)^c$, $n \in \mathbb{N}^*$. We have the partition $\bigcup_{n \in \mathbb{N}^*} M_n = M$, and

$$\begin{aligned} E_P[|\Lambda_F|\ f \circ T] \quad &= \quad \sum_{n=0}^{\infty} E_P[1_{M_n} |\Lambda_F|\ f \circ T] \\ &= \quad \sum_{n=0}^{\infty} E_P\left[1_{T(M_n)} f\right] = E_P[f N(x; M)]. \end{aligned}$$

The computation of $d(I_B + F)_* P_{|M}/dP$ follows from

$$E_P[1_M f \circ T] = \sum_{n=0}^{\infty} E_P\left[1_{T(M_n)} \frac{f}{\Lambda_F \circ T}\right] = E_P\left[f \sum_{\theta \in T^{-1}(x) \bigcap M} \frac{1}{\Lambda_F(\theta)}\right].$$

□

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