A pointwise equivalence of gradients on configuration spaces

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Abstract - *This Note aims to provide a short and self-contained proof of an equivalence between square field operators on configuration spaces. A recent result in analysis on configuration spaces, namely an equivalence between Dirichlet forms, is retrieved as a particular case. Our method relies on duality formulas and on extensions of the stochastic integral on Poisson space.*

Une équivalence de gradients ponctuelle sur les espaces de configurations

Résumé - Cette Note donne une preuve concise d'une équivalence entre opérateurs carré du champ sur l'espace des configurations. Ceci permet de retrouver un résultat récent relatif à une équivalence entre formes de Dirichlet sur ce même espace. La méthode utilisée repose sur des formules de dualité et sur les extensions de l'intégrale stochastique sur l'espace de Poisson.

Version française abrégée

L'analyse sur les espaces de configurations développée dans [1] repose sur une égalité de normes en espérance pour le gradient local ∇^Υ et l'opérateur de différence finie D sur les espaces des configurations Υ^X sur une variété Riemannienne X. Dans cette Note nous présentons une version ponctuelle de cette relation, avec une preuve concise qui fait apparaître le rôle de la dualité sur l'espace de Poisson. On montre que

$$(\nabla^\Upsilon F, \nabla^\Upsilon G)_{L^2_\gamma(TX)} \quad = \quad \delta^-\left((\nabla^X DF, \nabla^X DG)_{TX}\right) + (\nabla^X DF, \nabla^X DG)_{L^2_\sigma(TX)} \tag{1}$$

pour $\gamma \in \Upsilon^X$ et $F, G \in \mathcal{S}$, où δ^- est l'adjoint de l'opérateur de différence finie D sous la mesure de Poisson π_σ d'intensité σ sur Υ^X. Cette relation s'écrit

$$(\nabla^\Upsilon F, \nabla^\Upsilon G)_{L^2_\gamma(TX)} = \int_X \left[(\nabla^X DF, \nabla^X DG)_{T_xX}\right](\gamma\backslash\{x\})d\gamma(x),$$

où $\gamma\backslash\{x\}$ représente la configuration $\gamma \in \Upsilon^X$ à laquelle on a enlevé le point $x \in X$. En prenant l'espérance dans (1) on obtient comme corollaire l'égalité

$$E_{\pi_\sigma}\left[(\nabla^\Upsilon F, \nabla^\Upsilon G)_{L^2_\gamma(TX)}\right] = E_{\pi_\sigma}\left[(\nabla^X DF, \nabla^X DG)_{L^2_\sigma(TX)}\right],$$

pour toutes fonctionnelles F et G régulières sur Υ^X, c'est à dire le Th. 5-2 de [1], et ce uniquement sous la mesure de Poisson π_σ. Plus généralement on montre que l'opérateur carré du champ associé à la seconde quantification différentielle d'un opérateur de diffusion conservatif H^X sur $\mathcal{C}_c^\infty(X)$, de carré du champ local Γ^X, est donné par

$$\Gamma^\Upsilon(F,G) = \delta^-\left(\Gamma^X(DF, DG)\right) + \int_X \Gamma^X(DF,DG)d\sigma, \quad F, G \in \mathcal{S}. \tag{2}$$

1 Introduction

The study of the stochastic calculus of variations for point processes started in [3], [2], has been developed in several different directions, using mainly two different gradients. The gradient on Fock space is interpreted as a difference operator and has been used in e.g. [7], [9], [11]. On the other hand, infinitesimal perturbations of configurations, cf. [3], [2], [6], [4], [1], lead to a gradient that defines a local Dirichlet form. Stochastic analysis on configuration spaces, cf. [1], relies on the equivalence of expectation norms (13) that links the local gradient ∇^Υ to the finite difference operator D on the configuration space Υ^X of a Riemannian manifold X. In this paper we use the duality on Poisson space to obtain a pointwise equivalence of gradients.

2 Difference operators on configuration spaces

The results of this section and the following hold without referring to a particular probability measure on Υ^X. Let X be a metric space and let the configuration space Υ^X on X be the set of Radon measures on X of the form $\sum_{i=1}^{i=N} \varepsilon_{x_i}$, with $(x_i)_{i=1}^{i=N} \subset X$, $x_i \neq x_j$, $\forall i \neq j$, and $N \in \mathbb{N} \cup \{\infty\}$. Let σ be a diffuse Borel measure on X. As a convention we identify $\gamma \in \Upsilon^X$ with its support. Following [10], for any $x \in X$ and any mapping $F : \Upsilon^X \longrightarrow \mathbf{R}$ we define the mappings $\varepsilon_x^+ F : \Upsilon^X \longrightarrow \mathbf{R}$ and $\varepsilon_x^- F : \Upsilon^X \longrightarrow \mathbf{R}$ by

$$\left[\varepsilon_x^- F\right](\gamma) = F(\gamma \backslash \{x\}), \quad \text{and} \quad \left[\varepsilon_x^+ F\right](\gamma) = F(\gamma \cup \{x\}), \quad \gamma \in \Upsilon^X.$$

The difference operator D has been defined in [10] as $D_x F = \varepsilon_x^+ F - \varepsilon_x^- F$, $x \in X$. For $u : \Upsilon^X \times X \longrightarrow \mathbf{R}$ measurable (and everywhere defined), the negative and positive Skorohod integral operators are defined as

$$\delta^-(u) = \int_X \varepsilon_x^- u_x d(\gamma(x) - \sigma(x)) = \int_X \varepsilon_x^- u_x d\gamma(x) - \int_X u_x d\sigma(x), \tag{3}$$

and

$$\delta^+(u) = \int_X \varepsilon_x^+ u_x (d\gamma(x) - d\sigma(x)) = \int_X u_x d\gamma(x) - \int_X \varepsilon_x^+ u_x d\sigma(x), \tag{4}$$

$\gamma \in \Upsilon^X$, provided the series and integrals converge. The proof of the following lemma is straightforward from (3) and (4).

Lemma 1 *For $u : \Upsilon^X \times X \longrightarrow \mathbf{R}$ measurable, the operators δ^-, δ^+ and D are linked to the anticipating pathwise integral by the identities*

$$\int_X u_x d(\gamma(x) - \sigma(x)) = \delta^+(u) + \int_X D_x u_x d\sigma(x), \tag{5}$$

and

$$\int_X u_x d(\gamma(x) - \sigma(x)) = \delta^-(u) + \int_X D_x u_x d\gamma(x), \tag{6}$$

provided that all series and integrals converge.

With respect to multiple integrals it is well known that D acts by annihilation and δ^- acts by creation. This property does not hold for δ^+, in fact $\delta^+(u) = \delta^-(u + Du)$.

3 Differential operators on configuration spaces

We now assume that X is a Riemannian manifold with volume element m, and $\sigma(dx) = \rho(x)m(dx)$ with $\rho \in L^2_{loc}(X,m)$. Let T_xX denote the tangent space at $x \in X$, let ∇^X denote the gradient on X, and let $L^2_\gamma(TX) = L^2(X,TX,\gamma)$, resp. $L^2_\sigma(TX) = L^2(X,TX,\sigma)$ denote the "random", resp. "deterministic" tangent space to Υ^X at $\gamma \in \Upsilon^X$, cf. [1]. Let also

$$\mathcal{S} = \left\{ f\left(\int_X \varphi_1 d\gamma, \ldots, \int_X \varphi_n d\gamma \right), \quad \varphi_1, \ldots, \varphi_n \in \mathcal{C}_c^\infty(X),\ f \in \mathcal{C}_b^\infty(\mathbf{R}^n),\ n \in \mathbf{N} \right\},$$

where $\mathcal{P}(\mathbf{R}^n)$ denotes the space of real polynomials in n variables, and

$$\mathcal{U} = \left\{ \sum_{i=1}^{i=n} F_i u_i \ :\ u_1, \ldots, u_n \in \mathcal{C}_c^\infty(X),\ F_1, \ldots, F_n \in \mathcal{S},\ n \geq 1 \right\}.$$

The following gradient has been defined for $X = \mathbf{R}^d$ in [2], p. 152, and for X a Riemannian manifold in [1].

Definition 1 *For any $F = f\left(\int_X \varphi_1 d\gamma, \ldots, \int_X \varphi_n d\gamma\right) \in \mathcal{S}$, let*

$$\nabla^\Upsilon_x F(\gamma) = \sum_{i=1}^{i=n} \partial_i f\left(\int_X \varphi_1 d\gamma, \ldots, \int_X \varphi_n d\gamma \right) \nabla^X \varphi_i(x), \quad x \in X.$$

The following result gives the relation between ∇^Υ, D and the gradient ∇^X on X.

Lemma 2 *We have for $F \in \mathcal{S}$ and $\gamma \in \Upsilon^X$:*

$$\nabla^\Upsilon_x F = \varepsilon_x^- \nabla^X D_x F, \quad \gamma(dx) - a.e. \text{ and } \varepsilon_x^+ \nabla^\Upsilon_x F = \nabla^X D_x F, \quad \sigma(dx) - a.e.$$

Proof. Let $F = f\left(\int_X \varphi_1 d\gamma, \ldots, \int_X \varphi_n d\gamma\right)$, $x \in X$, $\gamma \in \Upsilon^X$. If $x \in \gamma$, then

$$\begin{aligned}
\nabla^\Upsilon_x F(\gamma) &= \sum_{i=1}^{i=n} \partial_i f\left(\int_X \varphi_1 d(\gamma\backslash\{x\}) + \varphi_1(x), \ldots, \int_X \varphi_n d(\gamma\backslash\{x\}) + \varphi_n(x) \right) \nabla^X \varphi_i(x) \\
&= \nabla^X f\left(\int_X \varphi_1 d(\gamma\backslash\{x\}) + \varphi_1(x), \ldots, \int_X \varphi_n d(\gamma\backslash\{x\}) + \varphi_n(x) \right) \\
&= \nabla^X \varepsilon_x^+ f\left(\int_X \varphi_1 d(\gamma\backslash\{x\}), \ldots, \int_X \varphi_n d(\gamma\backslash\{x\}) \right) \\
&= \left(\nabla^X \varepsilon_x^+ F\right)(\gamma\backslash\{x\}) = \left(\nabla^X D_x F\right)(\gamma\backslash\{x\}), \quad \gamma(dx) - a.e.
\end{aligned}$$

If $x \notin \gamma$, hence σ(dx)-a.e., we have

$$\begin{aligned}
\varepsilon_x^+ \nabla^\Upsilon_x F(\gamma) &= \nabla^X f\left(\int_X \varphi_1 d\gamma + \varphi_1(x), \ldots, \int_X \varphi_n d\gamma + \varphi_n(x) \right) \\
&= \nabla^X \varepsilon_x^+ f\left(\int_X \varphi_1 d\gamma, \ldots, \int_X \varphi_n d\gamma \right) = \nabla^X \varepsilon_x^+ F(\gamma) = \nabla^X D_x F(\gamma). \quad \square
\end{aligned}$$

Lemma 2 and (3), (4), imply the following

Proposition 1 *For $F, G \in \mathcal{S}$ and any $\gamma \in \Upsilon^X$, we have the relations*

$$(\nabla^\Upsilon F, \nabla^\Upsilon G)_{L^2_\gamma(TX)} = \delta^-\left((\nabla^X DF, \nabla^X DG)_{TX} \right) + (\nabla^X DF, \nabla^X DG)_{L^2_\sigma(TX)}, \tag{7}$$

and

$$(\nabla^\Upsilon F, \nabla^\Upsilon G)_{L^2_\gamma(TX)} = \delta^+\left((\nabla^\Upsilon F, \nabla^\Upsilon G)_{TX} \right) + (\nabla^X DF, \nabla^X DG)_{L^2_\sigma(TX)}. \tag{8}$$

4 Integration by parts characterization

Theorem 1 *Let π be a probability measure on Υ^X under which $\delta^-(h)$, resp. $\delta^+(h)$, is integrable, $\forall h \in \mathcal{C}_c(X)$. The following statements are equivalent, and they hold if and only if π is the Poisson measure π_σ with intensity σ:*

$$E_\pi\left[F\delta^-(h)\right] = E_\pi\left[(DF, h)_{L^2(X,\sigma)}\right], \quad \forall\ h \in \mathcal{C}_c(X), \quad F \in \mathcal{S}, \tag{9}$$

$$E_\pi\left[F\delta^+(h)\right] = E_\pi\left[(DF, h)_{L^2(X,\gamma)}\right], \quad \forall\ h \in \mathcal{C}_c(X), \quad F \in \mathcal{S}, \tag{10}$$

$$E_\pi\left[\delta^+(u)\right] = 0, \quad \forall\ u \in \mathcal{U}, \tag{11}$$

$$E_\pi\left[\delta^-(u)\right] = 0, \quad \forall\ u \in \mathcal{U}. \tag{12}$$

Proof. Clearly, these statements are equivalent, due to Lemma 1 applied to $u = hF$, and to the relations $\delta^+(u) = \delta^-(\varepsilon^+_\cdot u_\cdot)$, $\delta^-(u) = \delta^+(\varepsilon^-_\cdot u_\cdot)$, $u \in \mathcal{U}$. Assume that (9) holds, let $h \in \mathcal{C}_c(X)$ and $\psi(z) = E_\pi[\exp\left(iz\int_X hd\gamma\right)]$, $z \in \mathbf{R}$. We have

$$\begin{aligned}
\frac{d}{dz}\psi(z) = iE_\pi &\left[\int_X hd\gamma \exp\left(iz\int_X hd\gamma\right)\right] \\
= \quad iE_\pi &\left[\delta^-(h)\exp\left(iz\int_X hd\gamma\right)\right] + iE_\pi\left[\int_X hd\sigma \exp\left(iz\int_X hd\gamma\right)\right] \\
= \quad iE_\pi &\left[\left(h, D\exp\left(iz\int_X hd\gamma\right)\right)_{L^2(X,\sigma)}\right] + i\psi(z)\int_X hd\sigma \\
= \quad i(h, &e^{izh} - 1)_{L^2(X,\sigma)} E_\pi\left[\exp\left(iz\int_X hd\gamma\right)\right] + i\psi(z)\int_X hd\sigma = i\psi(z)(h, e^{izh})_{L^2(X,\sigma)},
\end{aligned}$$

where we used the relation $D_x \exp(\int_X hd\gamma) = (e^{h_x} - 1)\exp(\int_X hd\gamma)$, $\sigma(dx)$-a.e. With the initial condition $\psi(0) = 1$, this gives $\psi(z) = \exp\int_X(e^{izh} - 1)d\sigma$, $z \in \mathbf{R}$. Conversely, this calculation shows by completeness of exponential functions that (9) is satisfied if $\pi = \pi_\sigma$. $\square$

Relations (11) and (12) can be regarded as the characterization results of [8], used respectively in [12] and [1]. Relation (9), resp. (10), has been obtained in [7], [9], resp. in [10], under the assumption $\pi = \pi_\sigma$. As a consequence, Th. 5-2 of [1] is obtained by taking expectations under π_σ in (7) or (8):

$$E_{\pi_\sigma}\left[(\nabla^\Upsilon F, \nabla^\Upsilon G)_{L^2_\gamma(TX)}\right] = E_{\pi_\sigma}\left[(\nabla^X DF, \nabla^X DG)_{L^2_\sigma(TX)}\right], \tag{13}$$

hence the generator of the diffusion process associated to the Dirichlet form given by ∇^Υ is the differential second quantization $d\Gamma(H^X_\sigma)$ of the Dirichlet operator $H^X_\sigma = \mathrm{div}^X_\sigma \nabla^X$, cf. [1], and [14], [2] for the construction of this process.

Remark 1 *Although (8) gives (13) by taking expectations, it does not yield any information as a random identity. On the other hand, (7) is a true pointwise identity that also reads*

$$(\nabla^\Upsilon F, \nabla^\Upsilon G)_{L^2_\gamma(TX)} = \int_X \left[(\nabla^X DF, \nabla^X DG)_{T_xX}\right](\gamma\backslash\{x\})d\gamma(x), \quad \gamma \in \Upsilon^X.$$

5 Second quantization and carré du champ operators

In this section we work in a more general setting and show that (13) is a consequence of the computation of the carré du champ operator associated to second quantized diffusion operators. The difference with the above construction is that we are not restricted to the structure given by ∇^X. We refer to [13] and [5] for the definition of diffusion operators, carré du champ operators and their locality property on the Wiener and Poisson spaces as well as on more general structures. The differential second quantization $d\Gamma(H)$ of an operator H^X on $\mathcal{C}_c^\infty(X)$ is defined as $d\Gamma(H^X)F = \delta^-(H^X DF)$, $F \in \mathcal{S}$. The following theorem shows how to compute the carré du champ of $d\Gamma(H^X)$.

Theorem 2 *Let H^X be a diffusion operator on $\mathcal{C}_c^\infty(X)$, conservative under σ, with carré du champ Γ^X. Then $d\Gamma(H^X)$ is a diffusion operator with carré du champ*

$$\Gamma^\Upsilon(F,G) = \delta^-\left(\Gamma^X(DF,DG)\right) + \int_X \Gamma^X(DF,DG)d\sigma, \quad F,G \in \mathcal{S}. \tag{14}$$

Proof. Defining $\Gamma^\Upsilon(F,G)$ by (14) we have, using (5) and the relations $\delta^+(u) = \delta^-(u+Du)$, $DF^2 = 2FDF + DFDF$ σ-a.e.:

$$\begin{aligned}
&\frac{1}{2}d\Gamma(H^X)(F^2) = \frac{1}{2}\delta^-\left(H^X D(F^2)\right) = \delta^-\left(FH^X DF + H^X(DFDF)\right)\\
&= \quad Fd\Gamma(H^X)F - (DF, H^X DF)_{L^2(X,\sigma)} + \delta^-\left(\Gamma^X(DF,DF)\right) = Fd\Gamma(H^X)F + \Gamma^\Upsilon(F,F).
\end{aligned}$$

It remains to prove that Γ^Υ is local. We have

$$\begin{aligned}
&\delta^-\left(\Gamma^X(DF^2,DG)\right) + \int_X \Gamma^X(DF^2,DG)d\sigma = 2\delta^-\left(F\Gamma^X(DF,DG)\right)\\
&\qquad +\delta^-\left(\Gamma^X(DFDF,DG)\right) + 2F\int_X \Gamma^X(DF,DG)d\sigma + \int_X \Gamma^X(DFDF,DG)d\sigma\\
&= \quad 2\delta^-\left(F\Gamma^X(DF,DG)\right) + 2\delta^-\left(DF\Gamma^X(DF,DG)\right)\\
&\qquad +2F\int_X \Gamma^X(DF,DG)d\sigma + 2(DF,\Gamma^X(DF,DG))_{L^2(X,\sigma)} = 2F\Gamma^X(DF,DG). \quad \square
\end{aligned}$$

Relation (7) and Th. 2 show in particular that $(F,G) \mapsto (\nabla^\Upsilon F, \nabla^\Upsilon G)_{L^2_\gamma(TX)}$ is the carré du champ operator associated to the differential second quantization $d\Gamma(H_\sigma^X)$ of H_σ^X.

6 Stochastic integration

We now work with the generator H_σ^X and carré du champ Γ_σ^X given by ∇^X and study the consequences of Th. 2 on stochastic integration (H_σ^X is assumed to be conservative). We state a definition of adaptedness that does not require an ordering on X, following the Wiener space construction of [15]. Let $\mathcal{V} = \{\nabla^X u \ : \ u \in \mathcal{U}\}$. A vector $u = \sum_{i=1}^{i=n} F_i \nabla^X u_i \in \mathcal{V}$ is said to be adapted if $u_i DF_i = 0$ on $X \times \Upsilon^X$, $i = 1,\ldots,n$. The set of square-integrable adapted vectors, denoted by $L^2_{ad}(\Upsilon^X \times X, TX)$ is the completion in $L^2(\Upsilon^X, \pi_\sigma) \otimes L^2(X, TX, \sigma)$ of the adapted vectors in $\mathcal{V}$. For $x \in X$ we define the bilinear form trace_x on $T_xX \otimes T_xX$ by

$$\mathrm{trace}_x u \otimes v = (u,v)_{T_xX}, \qquad u \otimes v \in T_xX \otimes T_xX.$$

Let π be a probability on Υ^X under which ∇^Υ admits an adjoint on $\mathcal{S} \times \mathcal{V}$, denoted by $\mathrm{div}_\pi^\Upsilon$, and such that $\mathcal{S}$ is dense in $L^2(\Upsilon^X, \pi)$.

Proposition 2 *We have for* $u = \sum_{i=1}^{i=n} G_i \nabla^X u_i \in \mathcal{V}$:

$$\mathrm{div}_\pi^\Upsilon u = \sum_{i=1}^{i=n} G_i \mathrm{div}_\pi^\Upsilon \nabla^X u_i - \int_X \mathrm{trace}_x \nabla_x^\Upsilon u(x) \gamma(dx). \qquad (15)$$

If π *is the Poisson measure* π_σ *with intensity* σ*, then*

$$\mathrm{div}_{\pi_\sigma}^\Upsilon u = \int_X \mathrm{div}_\sigma^X u(x)\gamma(dx) - \int_X \mathrm{trace}_x \nabla_x^\Upsilon u(x)\gamma(dx), \quad u \in \mathcal{U}, \qquad (16)$$

and for any $u \in L^2_{ad}(\Upsilon^X \times X, TX)$,

$$\mathrm{div}_{\pi_\sigma}^\Upsilon (u) = \delta^-(\mathrm{div}_\sigma^X u) = \int_X \mathrm{div}_\sigma^X u(x)\gamma(dx). \qquad (17)$$

Proof. Relation (15) follows from the derivation property of ∇^Υ. Relation (16) holds because $\mathrm{div}_{\pi_\sigma}^\Upsilon \nabla^X u_i = \delta^-(\mathrm{div}_\sigma^X \nabla^X u_i) = \int_X \mathrm{div}_\sigma^X \nabla^X u_i d\gamma$ from (13). Relation (17) follows from (16) and the definition of adaptedness, using a density argument and the Itô isometry.

References

[1] S. Albeverio, Yu. G. Kondratiev, and M. Röckner. Analysis and geometry on configuration spaces. *J. Funct. Anal.*, 154:444–500, 1998.

[2] K. Bichteler, J.B. Gravereaux, and J. Jacod. *Malliavin Calculus for Processes with Jumps*, volume 2 of *Stochastics Monographs.* Gordon and Breach, 1987.

[3] J.M. Bismut. Calcul des variations stochastique et processus de sauts. *Zeitschrift für Wahrscheinlichkeitstheories Verw. Gebiete*, 63:147–235, 1983.

[4] N. Bouleau. Constructions of Dirichlet structures. In J. Kral et al., editor, *Potential theory – ICPT '94, Kouty, Czech Republic, August 13–20, 1994*, pages 9–25. de Gruyter, 1996.

[5] N. Bouleau and F. Hirsch. *Dirichlet Forms and Analysis on Wiener Space.* de Gruyter, 1991.

[6] E. Carlen and E. Pardoux. Differential calculus and integration by parts on Poisson space. In *Stochastics, Algebra and Analysis in Classical and Quantum Dynamics*, pages 63–73. Kluwer, 1990.

[7] A. Dermoune, P. Krée, and L. Wu. Calcul stochastique non adapté par rapport à la mesure de Poisson. In *Séminaire de Probabilités XXII*, volume 1321 of *Lecture Notes in Mathematics.* Springer Verlag, 1988.

[8] J. Mecke. Stationäre zufällige Masse auf lokalkompakten Abelschen Gruppen. *Z. Wahrscheinlichkeitstheorie Verw. Geb.*, 9:36–58, 1967.

[9] D. Nualart and J. Vives. A duality formula on the Poisson space and some applications. In E. Bolthausen, M. Dozzi, and F. Russo, editors, *Seminar on Stochastic Analysis, Random Fields and Applications*, Progress in Probability, pages 205–213, Ascona, 1993. Birkäuser.

[10] J. Picard. Formules de dualité sur l'espace de Poisson. *Ann. Inst. H. Poincaré Probab. Statist.*, 32(4):509–548, 1996.

[11] J. Picard. On the existence of smooth densities for jump processes. *Probab. Theory Related Fields*, 105:481–511, 1996.

[12] S. Roelly and H. Zessin. Une caractérisation de champs gibbsiens sur un espace de trajectoires. *C. R. Acad. Sci. Paris Sér. I Math.*, 321:1377–1382, 1995.

[13] D. Stroock. The Malliavin calculus, a functional analytic approach. *J. Funct. Anal.*, 44:212–257, 1981.

[14] D. Surgailis. On multiple Poisson stochastic integrals and associated Markov semi-groups. *Probability and Mathematical Statistics*, 3:217–239, 1984.

[15] A.S. Üstünel. Construction du calcul stochastique sur un espace de Wiener abstrait. *C. R. Acad. Sci. Paris Sér. I Math.*, 305:279–282, 1987.