(Probabilités)

Calculus on Fock space and a non-adapted quantum Itô formula

Nicolas Privault

Abstract - *The aim of this note is to introduce a calculus on Fock space with its probabilistic interpretations, and to give a detailed presentation of the associated quantum Itô formula.*

Calcul sur l'espace de Fock et une formule d'Itô non-commutative anticipante

Résumé - Le but de cette note est d'introduire un calcul sur l'espace de Fock et de donner une présentation détaillée de la formule d'Itô non-commutative associée.

Version française abrégée - Soit $\Phi = \bigoplus_{n\geq 0} H^{\circ n}$ l'espace de Fock symétrique sur $H = L^2(\mathbb{R}_+)$ avec ses opérateurs ∇, ∇^* d'annihilation et de création. Soit $\mathcal{S}$ le sous-ensemble de Φ dont les éléments ont un développement fini qui ne fait intervenir que des fonctions $\mathcal{C}_c^1$, et soit $\mathcal{U}$ l'ensemble des éléments de $\Phi \otimes L^2(\mathbb{R}_+)$ de la forme $\sum_{i=1}^{i=n} F_i \otimes h_i$, avec $h_1, \dots, h_n \in \mathcal{C}_c^1(\mathbb{R}_+)$, et $F_1, \dots, F_n \in \mathcal{S}$, $n \in \mathbb{N}$. Pour $h \in L^2(\mathbb{R}_+)$, on pose $\overset{\circ}{h}(t) = \int_0^t h(s)ds$ et $h_{[t} = 1_{[t,\infty[}h$, $t \in \mathbb{R}_+$.

Définition *On définit les opérateurs linéaires* $\tilde{\nabla} : \Phi \to \Phi \otimes L^2(\mathbb{R}_+)$ *sur* $\mathcal{S}$ *et* $\tilde{\nabla}^* : \Phi \otimes L^2(\mathbb{R}_+) \to \Phi$ *sur* $\mathcal{U}$ *par*

$$\tilde{\nabla}_t f^{\circ n} = -n f'_{[t} \circ f^{\circ(n-1)}, \quad t \in \mathbb{R}_+, \ f \in \mathcal{C}_c^1(\mathbb{R}_+), \quad n \in \mathbb{N},$$

$$\tilde{\nabla}^*(f^{\circ n} \otimes h) = n \left(f\overset{\circ}{h} \right)' \circ f^{\circ(n-1)}, \quad f, h \in \mathcal{C}_c^1(\mathbb{R}_+),$$

et par polarisation.

Les opérateurs $\tilde{\nabla}$, $\tilde{\nabla}^*$ sont fermables et $\tilde{\nabla}^* : \Phi \otimes L^2(\mathbb{R}_+) \to \Phi$ est l'adjoint de $\tilde{\nabla} : \Phi \to \Phi \otimes L^2(\mathbb{R}_+)$. Soit $(W, L^2(\mathbb{R}_+), \mu)$ l'espace de Wiener avec le mouvement brownien $(B_t)_{t\in\mathbb{R}_+}$. Pour $h \in L^2(\mathbb{R}_+)$ avec $\sup_{x\in\mathbb{R}_+} \mid h(x) \mid < 1$, soit $\nu_h(t) = t + \int_0^t h(s)ds$, $t \in \mathbb{R}_+$. On définit le changement de temps $\mathcal{T}_h : W \to W$ par (1). Sous l'identification de Wiener usuelle entre Φ et $L^2(W, \mu)$,

$$\int_0^\infty h(t) \left(\tilde{\nabla}_t + \frac{1}{2} \nabla_t \nabla_t \right) F dt = - \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} (F \circ \mathcal{T}_{\varepsilon h} - F), \quad F \in \mathcal{S}.$$

Dans l'interprétation poissonienne de Φ, $\nabla + \tilde{\nabla}$ coincide avec l'opérateur $\tilde{D}$ défini sur l'espace de Poisson par changement de temps (4). Soit $\tilde{\delta}$ l'adjoint de $\tilde{D}$, et soient $(\tilde{a}_t^\pm)_{t\in\mathbb{R}_+}$ les processus d'opérateurs définis par

$$\tilde{a}_t^- F = \int_0^t \tilde{D}_s F ds, \quad \tilde{a}_t^+ F = \tilde{\delta}(1_{[0,t]} F), \quad F \in \mathcal{S}, \ t \in \mathbb{R}_+.$$

Comme le montre la définition de $\tilde{\nabla}$, $(\tilde{a}_t^\pm)_{t\in\mathbb{R}_+}$ sont des perturbations des processus de création et d'annihilation sur Φ, et ne sont pas adaptés en tant que processus d'opérateurs. Par conséquent $\tilde{a}_t^\pm$ ne commute pas avec sa différentielle, et la formule d'Itô de [1] doit donc être écrite comme

Proposition *Si* X, Y *sont des processus simples adaptés tels que* $\Xi \subset Dom(X_s Y_s)$, $s \in \mathbb{R}_+$*:*

$$\begin{aligned} \int_0^t X_s d\tilde{a}_s^\varepsilon \int_0^t Y_s d\tilde{a}_s^\eta &= \int_0^t d\tilde{a}_s^\varepsilon X_s \left(\int_0^s Y_u d\tilde{a}_u^\eta \right) + \int_0^t \left(\int_0^s X_u d\tilde{a}_u^\varepsilon \right) Y_s d\tilde{a}_s^\eta \\ &\quad + \int_0^t X_s Y_s d\tilde{a}_s^\varepsilon \cdot d\tilde{a}_s^\eta, \quad \varepsilon, \eta = -, +, \end{aligned}$$

où le produit $d\tilde{a}_s^- \cdot d\tilde{a}_s^+$ *vaut* $d\tilde{a}_t^- \cdot d\tilde{a}_t^+ = dN_t$*, les autres produits étant nuls.*

1 A calculus on Fock space

The annihilation operator $\nabla : \Phi \to \Phi \otimes H$ on the symmetric Fock space $\Phi = \bigoplus_{n\geq 0} H^{\circ n}$ over $H = L^2(\mathbb{R}_+)$ is defined by transformation of the tensor $f^{\circ n}$, $f \in H$, into $\nabla f^{\circ n} = n f^{\circ(n-1)} \otimes f$, $n \in \mathbb{N}$, while the creation operator $\nabla^* : \Phi \otimes H \to \Phi$ satisfies $\nabla^* f^{\circ n} \otimes g = f^{\circ n} \circ g$, $n \in \mathbb{N}$. The space Φ has two main probabilistic interpretations. In the Wiener interpretation, the sum of the creation and annihilation operators is identified to a Brownian motion which can be perturbed by the number operator process in order to give a Poisson process in the Poisson interpretation of Φ. The operators $\tilde{\nabla}$ and $\tilde{\nabla}^*$ defined below will be interpreted as perturbations of ∇ and ∇^*. For $f \in \mathcal{C}_c^1(\mathbb{R}_+)$, $h \in L^2(\mathbb{R}_+)$, let $\overset{\circ}{h}(t) = \int_0^t h(s)ds$, $h_{[t} = 1_{[t,\infty[}h$, and $f'(t) = \frac{d}{dt} f(t)$, $t \in \mathbb{R}_+$. Let $\mathcal{S}$ denote the subset of Φ whose elements have a finite development involving only $\mathcal{C}_c^1$ functions, and let $\mathcal{U}$ denote the set of elements of $\Phi \otimes L^2(\mathbb{R}_+)$ of the form $\sum_{i=1}^{i=n} F_i \otimes h_i$, with $h_1, \ldots, h_n \in \mathcal{C}_c^1(\mathbb{R}_+)$, and $F_1, \ldots, F_n \in \mathcal{S}$, $n \in \mathbb{N}$.

Definition 1 *We define the linear operators $\tilde{\nabla} : \Phi \to \Phi \otimes L^2(\mathbb{R}_+)$ on $\mathcal{S}$ and $\tilde{\nabla}^* : \Phi \otimes L^2(\mathbb{R}_+) \to \Phi$ on $\mathcal{U}$ by*

$$\tilde{\nabla}_t f^{\circ n} = -n f'_{[t} \circ f^{\circ(n-1)}, \quad t \in \mathbb{R}_+, \ f \in \mathcal{C}_c^1(\mathbb{R}_+), \quad n \in \mathbb{N},$$

$$\tilde{\nabla}^*(f^{\circ n} \otimes h) = n \left(f\overset{\circ}{h} \right)' \circ f^{\circ(n-1)}, \quad f, h \in \mathcal{C}_c^1(\mathbb{R}_+),$$

and by polarization of these expressions.

Both $\tilde{\nabla}$, $\tilde{\nabla}^*$ are closable, and the operator $\tilde{\nabla}^* : \Phi \otimes L^2(\mathbb{R}_+) \to \Phi$ is the adjoint operator of $\tilde{\nabla} : \Phi \to \Phi \otimes L^2(\mathbb{R}_+)$. The domain of $\tilde{\nabla}$ is determined by the equality of norms

$$\| \tilde{\nabla} F \|^2_{\Phi \otimes L^2(\mathbb{R}_+)} = \sum_{n \geq 1} n^3 \int_0^\infty t \, \| \partial_1 f_n(t, \cdot) \|^2_{L^2(\mathbb{R}_+)^{\circ(n-1)}} \, dt, \quad F \in \mathcal{S}, \ F = \sum_{n \in \mathbb{N}} f_n.$$

2 Probabilistic interpretations

Let $(W, L^2(\mathbb{R}_+), \mu)$ denote the classical Wiener space, with Brownian motion $(B_t)_{t \in \mathbb{R}_+}$. We recall that under the usual identification between Φ and $L^2(W, \mu)$, ∇ is identified to a derivation operator which satisfies

$$(\nabla F, h)_{L^2(\mathbb{R}_+)} = \lim_{\varepsilon \to 0} \frac{F\left(B_\cdot + \varepsilon \int_0^\cdot h(s)ds\right) - F}{\varepsilon}, \quad F \in \mathcal{S}, \ h \in L^2(\mathbb{R}_+),$$

cf. e.g. [2], [3], [4]. For $h \in L^2(\mathbb{R}_+)$, with $\sup_{x \in \mathbb{R}_+} | h(x) | < 1$, let $\nu_h(t) = t + \int_0^t h(s)ds$, $t \in \mathbb{R}_+$. We define a mapping $\mathcal{T}_h : W \to W$, $t, \varepsilon \in \mathbb{R}_+$, as

$$\mathcal{T}_h(\omega) = \omega \circ \nu_h^{-1}, \quad h \in L^2(\mathbb{R}_+), \ \sup_{x \in \mathbb{R}_+} | h(x) | < 1. \tag{1}$$

Although $\mathcal{T}_h$ is not absolutely continuous on the Wiener space, the functional $F \circ \mathcal{T}_h$ is well-defined for $F \in \mathcal{S}$, since elements of $\mathcal{S}$ can be defined trajectory by trajectory.

Proposition 1 *On the Wiener space,* $\tilde{\nabla}$ *satisfies the relation*

$$\tilde{\nabla}_t(FG) = F\tilde{\nabla}_t G + G\tilde{\nabla}_t F - \nabla_t F \nabla_t G, \quad t \in \mathbb{R}_+, \quad F, G \in \mathcal{S}, \tag{2}$$

and

$$\int_0^\infty h(t) \left(\tilde{\nabla}_t + \frac{1}{2}\nabla_t \nabla_t\right) F dt = -\lim_{\varepsilon \to 0} \frac{1}{\varepsilon}(F \circ \mathcal{T}_{\varepsilon h} - F), \quad F \in \mathcal{S}, \quad h \in L^2(\mathbb{R}_+).$$

Let $(N_t)_{t\in\mathbb{R}_+}$ be a Poisson process on a probability space (B, P). From [5], [6], [7], ∇_t acts in the Poisson identification of $L^2(B, P)$ with Φ by perturbation of the Poisson process trajectory via addition of a jump at time t, and

$$\nabla(FG) = F\nabla G + G\nabla F + \nabla F \nabla G, \quad F, G \in \mathcal{S}. \tag{3}$$

A gradient operator $\tilde{D} : L^2(B) \to L^2(B) \otimes L^2(\mathbb{R}_+)$ on Poisson space, cf. [8], [9], is defined as

$$(\tilde{D}F, h)_{L^2(\mathbb{R}_+)} = -\lim_{\varepsilon \to 0} \frac{1}{\varepsilon}(F \circ \mathcal{T}_{\varepsilon h} - F), \quad F \in \mathcal{S}, \quad h \in L^2(\mathbb{R}_+), \tag{4}$$

where the transformation $\mathcal{T}_h$ is defined by the time change ν_h applied to $(N_t)_{t\in\mathbb{R}_+}$. Unlike in the Wiener space case, the transformation $\mathcal{T}_h$ is absolutely continuous in the Poisson space case. The operator $\tilde{D}$ is closable, its adjoint $\tilde{\delta}$ satisfies

$$\tilde{\delta}(u) = \int_0^\infty u(s) d(N_s - s) - \int_0^\infty \tilde{D}_s u(s) ds,$$

for $u(t) = f(t, T_1, \ldots, T_n)$, $f \in \mathcal{C}_c^1(\mathbb{R}_+^{n+1})$, and coincides with the compensated Poisson stochastic integral on the square-integrable adapted processes, cf. [8], [9].

Proposition 2 *On the Poisson space,* $\tilde{D} = \nabla + \tilde{\nabla}$*, and* $\tilde{\nabla}$ *also satisfies the relation*

$$\tilde{\nabla}_t(FG) = F\tilde{\nabla}_t G + G\tilde{\nabla}_t F - \nabla_t F \nabla_t G, \quad t \in \mathbb{R}_+, \ F, G \in \mathcal{S}.$$

In the Wiener interpretation of Φ, $\tilde{\delta}$ is identified to an extension $\nabla^* + \tilde{\nabla}^*$ of the stochastic integral with respect to $(B_t)_{t\in\mathbb{R}_+}$. As an application of this calculus, we obtain the following absolute continuity criterion for single Poisson stochastic integrals.

Proposition 3 *Let* $f \in L^2(\mathbb{R}_+)$ *such that* $\int_0^\infty t f'(t)^2 dt < \infty$ *and* $\{f' = 0\}$ *has finite Lebesgue measure. Then the law of* $\int_0^\infty f(t) d(N_t - t)$ *is absolutely continuous with respect to the Lebesgue measure.*

3 A non-adapted quantum Itô formula

Define the operator processes $(\tilde{a}_t^{\pm})_{t\in\mathbb{R}_+}$ as

$$\tilde{a}_t^- F = \int_0^t \tilde{D}_s F ds, \quad \tilde{a}_t^+ F = \tilde{\delta}(1_{[0,t]} F), \quad F \in \mathcal{S}, \ t \in \mathbb{R}_+.$$

In the Poisson interpretation of Φ, $\left(\tilde{a}_t^- + \tilde{a}_t^+\right)_{t\in\mathbb{R}_+}$ is a decomposition of the multiplication operator by the compensated Poisson process $(N_t - t)_{t\in\mathbb{R}_+}$. The aim of this section is to make a precise statement of a fact that has been overlooked in [1], [10]. Namely, the processes $(\tilde{a}_t^{\pm})_{t\in\mathbb{R}_+}$ are not adapted operator processes, as can be seen from the definition of $\tilde{\nabla}$. Consequently, $\tilde{a}_t^{\pm}$ does not commute with its differential. Let

$$\int_0^t d\tilde{a}_s^+ \tilde{a}_s^{\pm} F = \tilde{\delta}\left(1_{[0,t]}\tilde{a}_\cdot^{\pm} F\right), \quad \int_0^t d\tilde{a}_s^- \tilde{a}_s^{\pm} F = \int_0^t \tilde{D}_s \tilde{a}_s^{\pm} F ds, \quad F \in \mathcal{S}.$$

The integrals $\int_0^t \tilde{a}_s^{\pm} d\tilde{a}_s^+$ and $\int_0^t \tilde{a}_s^{\pm} d\tilde{a}_s^-$ are defined by duality on $\mathcal{S}$ as being distribution-valued in the dual $\mathbb{D}_{2,-1}$ of $Dom(\tilde{D})$. They are the respective adjoints of $\int_0^t d\tilde{a}_s^- \tilde{a}_s^{\mp}$, $\int_0^t d\tilde{a}_s^+ \tilde{a}_s^{\mp}$. Prop. 7 in [1] should read after reordering of the differentials:

Proposition 4 *We have for simple adapted processes X, Y such that $\Xi \subset Dom(X_s Y_s)$, $s \in \mathbb{R}_+$:*

$$\begin{aligned}\int_0^t X_s d\tilde{a}_s^{\varepsilon} \int_0^t Y_s d\tilde{a}_s^{\eta} &= \int_0^t d\tilde{a}_s^{\varepsilon} X_s \left(\int_0^s Y_u d\tilde{a}_u^{\eta}\right) + \int_0^t \left(\int_0^s X_u d\tilde{a}_u^{\varepsilon}\right) Y_s d\tilde{a}_s^{\eta} \\ &\quad + \int_0^t X_s Y_s d\tilde{a}_s^{\varepsilon} \cdot d\tilde{a}_s^{\eta}, \quad \varepsilon, \eta = -, +,\end{aligned}$$

where the product $d\tilde{a}_s^- \cdot d\tilde{a}_s^+$ is given by $d\tilde{a}_t^- \cdot d\tilde{a}_t^+ = dN_t$, the other products being zero.

This expression might not always be defined on all $F \in \mathcal{S}$, and we will prove in general the following relations:

$$\begin{aligned}
&< \int_0^t Y_s d\tilde{a}_s^+ G, \int_0^t X_s^* d\tilde{a}_s^- F > \\
&= \int_0^t < \int_0^s Y_u d\tilde{a}_u^+ G, X_s^* \tilde{D}_s F > ds + \int_0^t < Y_s G, \tilde{D}_s \int_0^s X_u^* d\tilde{a}_u^- F > ds, \\
&< \int_0^t Y_s d\tilde{a}_s^- G, \int_0^t X_s^* d\tilde{a}_s^+ F > \\
&= \int_0^t < \tilde{D}_s G, \int_0^s Y_u^* d\tilde{a}_u^+ X_s^* F > ds + \int_0^t < \tilde{D}_s \int_0^s X_u d\tilde{a}_u^- Y_s G, F > ds, \\
&< \int_0^t Y_s d\tilde{a}_s^+ G, \int_0^t X_s^* d\tilde{a}_s^+ F > \\
&= \int_0^t < \tilde{D}_s X_s \int_0^s Y_u d\tilde{a}_u^+ G, F > ds + \int_0^t < Y_s G, \int_0^s X_u^* d\tilde{a}_u^+ \tilde{D}_s F > ds, \\
&\quad + < \int_0^t X_s Y_s G dN_s, F >, \\
&< \int_0^t Y_s d\tilde{a}_s^- G, \int_0^t X_s^* d\tilde{a}_s^- F > \\
&= \int_0^t < \int_0^s Y_u d\tilde{a}_u^- G, X_s^* \tilde{D}_s F > ds + \int_0^t < Y_s \tilde{D}_s G, \int_0^s X_u^* d\tilde{a}_u^- F > ds,
\end{aligned}$$

for $F, G \in \mathcal{S}$. By non-commutative linearity and adaptedness of $(X_t)_{t\in\mathbb{R}_+}$, $(Y_t)_{t\in\mathbb{R}_+}$, they are a consequence of the following lemma, which is identical to Lemma 1 of [1], except for the reordering between $\tilde{a}_s^{\pm}$ and its differential.

Lemma 1 *We have*

$$\tilde{a}_t^+ \tilde{a}_t^- = \int_0^t d\tilde{a}_s^+ \tilde{a}_s^- + \int_0^t \tilde{a}_s^+ d\tilde{a}_s^-, \tag{5}$$

$$\tilde{a}_t^{\pm} \tilde{a}_t^{\pm} = \int_0^t \tilde{a}_s^{\pm} d\tilde{a}_s^{\pm} + \int_0^t d\tilde{a}_s^{\pm} \tilde{a}_s^{\pm}, \tag{6}$$

$$\tilde{a}_t^- \tilde{a}_t^+ = \int_0^t d\tilde{a}_s^- \tilde{a}_s^+ + \int_0^t \tilde{a}_s^- d\tilde{a}_s^+ + N_t. \tag{7}$$

Proof. We have for (5):

$$\begin{aligned} < \tilde{a}_t^- G, \tilde{a}_t^- F > &= < \int_0^t \tilde{D}_u G du, \int_0^t \tilde{D}_s F ds > \\ &= \int_0^t \int_0^s < \tilde{D}_u G, \tilde{D}_s F > duds + \int_0^t \int_0^u < \tilde{D}_u G, \tilde{D}_s F > duds \\ &= < \tilde{\delta} \left(1_{[0,t]}(\cdot) \int_0^\cdot \tilde{D}_u G du \right), F > + < G, \tilde{\delta} \left(1_{[0,t]}(\cdot) \int_0^\cdot \tilde{D}_u F du \right) > \\ &= < \int_0^t d\tilde{a}_s^+ \tilde{a}_s^- G, F > + < G, \int_0^t d\tilde{a}_s^+ \tilde{a}_s^- F > \\ &= < G, \left(\int_0^t \tilde{a}_s^+ d\tilde{a}_s^- + \int_0^t d\tilde{a}_s^+ \tilde{a}_s^- \right) F >, \quad F, G \in \mathcal{S}, \end{aligned}$$

and

$$\begin{aligned} &< \tilde{a}_t^+ F, \tilde{a}_t^- G > \\ &= \int_0^t < \tilde{a}_s^+ F, \tilde{D}_s G > ds + < F, \int_0^t \tilde{a}_s^- G d\tilde{N}_s > - \int_0^t < \tilde{D}_s F, \tilde{a}_s^- G > ds \\ &= \int_0^t < \tilde{a}_s^+ F, \tilde{D}_s G > ds + < F, \int_0^t d\tilde{a}_s^+ \tilde{a}_s^- G > + < F, \int_0^t d\tilde{a}_s^- \tilde{a}_s^- G > - \int_0^t < \tilde{D}_s F, \tilde{a}_s^- G > ds \\ &= \int_0^t < \tilde{a}_s^+ F, \tilde{D}_s G > ds + < F, \int_0^t d\tilde{a}_s^- \tilde{a}_s^- G > \\ &= < F, \int_0^t \tilde{a}_s^- d\tilde{a}_s^- G > + < F, \int_0^t d\tilde{a}_s^- \tilde{a}_s^- G >, \quad F, G \in \mathcal{S}, \end{aligned}$$

hence (6). Concerning (7),

$$\begin{aligned} < \tilde{a}_t^+ F, \tilde{a}_t^+ G > &= \int_0^t < \tilde{a}_s^+ F d\tilde{N}_s, G > - \int_0^t < \tilde{a}_s^+ F, \tilde{D}_s G > ds \\ &\quad + < F, \int_0^t \tilde{a}_s^+ G d\tilde{N}_s > - \int_0^t < \tilde{D}_s F, \tilde{a}_s^+ G > ds + < N_t F, G > \\ &= < \int_0^t d\tilde{a}_s^+ \tilde{a}_s^+ F, G > + < \int_0^t d\tilde{a}_s^- \tilde{a}_s^+ F, G > - \int_0^t < \tilde{a}_s^+ F, \tilde{D}_s G > ds \\ &\quad + < F, \int_0^t d\tilde{a}_s^+ \tilde{a}_s^+ G > + < F, \int_0^t d\tilde{a}_s^- \tilde{a}_s^+ G > \\ &\quad - \int_0^t < \tilde{D}_s F, \tilde{a}_s^+ G > ds + < N_t F, G > \\ &= < \int_0^t d\tilde{a}_s^- \tilde{a}_s^+ F, G > + < F, \int_0^t d\tilde{a}_s^- \tilde{a}_s^+ G > + < N_t F, G > \\ &= < \int_0^t d\tilde{a}_s^- \tilde{a}_s^+ F, G > + < \int_0^t \tilde{a}_s^- d\tilde{a}_s^+ F, G > + < N_t F, G >, \quad F, G \in \mathcal{S}. \end{aligned}$$

□

The commutation relation stated in [1], [10] now reads

$$[\tilde{a}_t^-, \tilde{a}_t^+] = \tilde{a}_t^- \tilde{a}_t^+ - \tilde{a}_t^+ \tilde{a}_t^- = N_t + \int_0^t \tilde{a}_s^- d\tilde{a}_s^+ - \int_0^t d\tilde{a}_s^+ \tilde{a}_s^- + \int_0^t d\tilde{a}_s^- \tilde{a}_s^+ - \int_0^t \tilde{a}_s^+ d\tilde{a}_s^-.$$

References

[1] N. Privault. A different quantum stochastic calculus for the Poisson process. *Probab. Theory Related Fields*, 105:255–278, 1996.

[2] D. Nualart. *The Malliavin Calculus and Related Topics*. Probability and its Applications. Springer-Verlag, 1995.

[3] A.S. Üstünel. *An introduction to analysis on Wiener space*, volume 1610 of *Lecture Notes in Mathematics*. Springer Verlag, 1995.

[4] S. Watanabe. *Lectures on Stochastic Differential Equations and Malliavin Calculus*. Tata Institute of Fundamental Research, 1984.

[5] A. Dermoune, P. Krée, and L. Wu. Calcul stochastique non adapté par rapport à la mesure de Poisson. In *Séminaire de Probabilités XXII*, volume 1321 of *Lecture Notes in Mathematics*, pages 477–484. Springer Verlag, 1988.

[6] Y. Ito. Generalized Poisson functionals. *Probab. Theory Related Fields*, 77:1–28, 1988.

[7] D. Nualart and J. Vives. Anticipative calculus for the Poisson process based on the Fock space. In *Séminaire de Probabilités XXIV*, volume 1426 of *Lecture Notes in Math.*, pages 154–165. Springer, Berlin, 1990.

[8] E. Carlen and E. Pardoux. Differential calculus and integration by parts on Poisson space. In S. Albeverio, Ph. Blanchard, and D. Testard, editors, *Stochastics, Algebra and Analysis in Classical and Quantum Dynamics (Marseille, 1988)*, volume 59 of *Math. Appl.*, pages 63–73. Kluwer Acad. Publ., Dordrecht, 1990.

[9] N. Privault. Chaotic and variational calculus in discrete and continuous time for the Poisson process. *Stochastics and Stochastics Reports*, 51:83–109, 1994.

[10] N. Privault. Une nouvelle représentation non-commutative du mouvement brownien et du processus de Poisson. *C. R. Acad. Sci. Paris Sér. I Math.*, 322:959–964, 1996.

Equipe d'Analyse et Probabilités, Université d'Evry
Boulevard des Coquibus, 91025 Evry Cedex, France