

Understanding Markov Chains - Examples and Applications

Errata to the Second (2018) Edition*

- Page 4 line -2: It should be “disjoint” instead of “disjoints”.
- Page 11 line 11: It should be $\frac{\partial F_X}{\partial x}(x)$ instead of $F_X(s)$.
- Page 18 line 7: It should be $0 + \mathbb{P}(A) = \mathbb{P}(A)$ on the right-hand side.
- Page 20 line 1: It should be $\mu/(\mu - \lambda)$ in the right-hand side.
- Page 20 line -7: Remove the division by $\mathbb{P}(A)$ on the right hand side.
- Page 28 line 18: It should be “ \mathcal{G} -measurable”, not “ \mathcal{H} -measurable”.
- Page 29 line 2: It should be “ \mathcal{H} -measurable”, not “ \mathcal{G} -measurable”.
- Page 41 line 7: Change “in the case” to “in the cases”.
- Page 41 line -10: The value of $f_3(1)$.
- Page 41 line -9: “step” should be “steps”.
- Page 43 line 2: $f(k)$ should be $f_S(k)$ in (2.2.6).
- Page 43 line 10: “rely” should be “relies”.
- Page 43 line -2: $1 \leq k \leq S - 1$ can be replaced with $1 < k \leq S - 1$.
- Page 46 lines 3 and -1: $f(k)$ should be $f_S(k)$.
- Page 51 line 15: $q \lim_{n \rightarrow \infty} (pq)^n = 0$.
- Page 51 line -6: (2.2.7) should be (2.2.27).
- Page 56 lines -13 to -8: $h(\cdot)$ should be $h_S(\cdot)$.
- Page 58 line -5: Delete “below”.
- Page 73, Figure 3.2: The blue segment from 12 to 13 can be removed.
- Page 74, Figure 3.3: Change “ $T_0^r = n$ ” to “ $n = 14$ ”.
- Page 77 line 5: “and $h(0) = 1$ ”.
- Page 77: The labels (3.4.10)-(3.4.12) should be moved two lines up.
- Page 79: The footnote 6 may be removed.
- Page 81: (13.9) should be (A.8).
- Page 82 line 16, Eq. (3.4.24): $\mathbb{P}(S_n = k \mid S_0 = 0)$ can be deleted.
- Page 92 line -7, Eq. (4.2.2): Change $i \in \mathbf{N}$ to $i \in \mathbb{S}$.
- Page 99 line 8: “matrix” should be “matrices”.

*With thanks to Moti Ben-Ari and Yung-Hsiang Huang.

- Page 105 line -1: $Z_n = 1$ should be $Z_n = i$.
- Page 116 line -3: g should be g_l .
- Page 117 line 12: Change “is said to be absorbing” into “is absorbing”.
- Page 121 line -10: “whenever $X_0 = k \in A$ ” should be “whenever $Z_0 = k \in A$ ”.
- Page 122 line 9: “Lemma 2.3” should be “Lemma 2.4”.
- Page 123 lines -4 and -2: The exponent should be “ $k - 1$ ”, not “ k ”.
- Page 124 line -8: “with the boundary condition $h_A(k) = f(k)$ ”.
- Page 124 lines 6, 7, 8 except for the second sum on line 8: It should be $\sum_{m=0}^N$ instead of $\sum_{m=0}^r$.
- Page 125 line 2: “with $h_{\{m\}}(m) = 0$ ”.
- Page 125 line 5: State $\textcircled{1}$ should be \textcircled{l} .
- Page 127 line -4: “which is in agreement with”.
- Page 133 lines 10-11: This duplicate sentence should be removed.
- Page 135 line -6: $p_{j,j}$ and $p_{i,j}$ can be replaced with p_{jj} and p_{ij} .
- Page 135 line -4: The mean number of returns to state \textcircled{j} , not \textcircled{i} .
- Page 135 line -2: ... and when $p_{ij} > 0$ it is finite, *i.e.* ...
- Page 136 line 10, $(I - P)^{-1}$ should be replaced with $(I - \tilde{P})^{-1}$, where $\tilde{P} := (P_{i,j})_{i \neq m, j \neq m}$.
- Page 137 line 8: The transition matrix should read

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \end{bmatrix} \end{matrix}.$$

- Page 139 line 3: It should be $X_n \in A$.
- Page 139 line 4: Remove the “ \subset ” sign.
- Page 143 Exercise 5.20: Add the Bellman equation $V^*(k) = R(k) + \gamma \max_{a \in \mathcal{A}} \sum_{l \in \mathbb{S}} P_{k,l}^a V^*(l)$ to the question.
- Page 144 line 1: Delete the first “Show, using first step analysis,”.
- Page 147, Remark 6.1, line 1: $\mathbb{1}_{\{i = j\}}$ should read $\mathbb{1}_{\{i=j\}}$.
- Page 148 lines 13 and 14: Remove “ $+\infty$ ” as upper index in the sum.
- Page 148 line -1: “to test the irreducibility”.
- Page 149 line 9: $P_{k,i} = 0, k \in \mathbb{S}$.

- Page 152 line -3: Change the sentence to “we find that a chain with finite state space cannot be transient as the matrix $I_d - P$ is not invertible in this case, see Theorem 6.9 below.”
- Page 153 line 13: $g_i(j) = \mathbb{P}(T_i^r < \infty \mid X_0 = j)$.
- Page 154 line -1: It should be $p_{1,1} = \mathbb{P}(T_1^r < \infty \mid X_0 = 1)$.
- Page 165 line -5: Starting from state \textcircled{i} .
- Page 174 line 10: Remove “to” in “at to”.
- Page 179 line 3: Change the index j to k on the right, i.e. $\frac{\pi_j P_{j,i}}{\pi_i P_{i,j}}$ becomes $\frac{\pi_k P_{k,i}}{\pi_i P_{i,k}}$.
- Page 180 line -5: The numerator of $P_{k,k}$ should be $2k(N - k)$.
- Page 184 line 14: Exercise 7.17: Add “ $P =$ ” before the matrix.
- Page 191 line -8: Delete “The next”.
- Page 192 line -5: Change “independent random variables” into “independent identically distributed random variables”.
- Page 193 line 1: Change $X_n^{(k)}$ to $X_n^{(l)}$ in the summation over l .
- Page 195 line 2: Add “2” in front of “ $\mathbb{P}(Y_1 \geq 2)$ ”.
- Page 195 line -8: Change “ $\sigma^2 \mu^n + \mu^2 \sigma_n^2$ ” to “ $\sigma^2 \mu^n + \mu \sigma_n^2$ ”.
- Page 196 line -10: It should be $\mathbb{P}(X_n = 0 \mid X_0 = 1) - \mathbb{P}(X_{n-1} = 0 \mid X_0 = 1)$.
- Page 198 line -11: Remove “(5.1.5) and”.
- Page 202 line 5: Replace the dot by a comma after “(sub)critical case”.
- Page 202 line -3: Change “parameter 1/2” into “parameter p ”.
- Page 208 line 4: Remove “determine”.
- Page 209 line 6: It should be “(h)”.
- Page 213 line -8: “(14.4) and (14.5)” should be “(B.4) and (B.5)”.
- Page 215 line 15: Change “strictly larger that” into “strictly larger than”.
- Page 220 line 5: Replace “-1” with “-1/h” on the right.
- Page 220 line 6: Divide both terms by h .
- Page 220 line -2: Remove “only” and change “can have” to “has”.
- Page 221 line -9: “with”, not “which”.
- Page 224 line 4: Remove “ $+\infty$ ” as upper index in the sum.
- Page 228, insert “The equation” before (9.4.3).
- Page 243 line -2: It should be “ $e^{t\lambda_n}$ ” (with no minus sign).
- Page 248 line 2: “and $\mu_1 > 0$ ”.

- Page 252 line -1: “When $\lambda = \mu$, Relation (2.2.12) shows that”.
- Page 258 Exercise 9.10: Change $0, 1, \dots, N$ to $\{0, 1, \dots, N\}$.
- Page 260 line 11, Exercise 9.17: Remove “the”.
- Page 260 line 19, Exercise 9.17: Delete “Note that the return time”.
- Page 264 line 8: “generated by $(X_n)_{n \geq 1}$ ” (not by $(S_n)_{n \in \mathbb{N}}$).
- Page 264 line 15: Delete “e.g.”.
- Page 266 line 14: occurred.
- Page 268 line -1: Change “ $\mathcal{F}_{l-1} \subset \mathcal{F}_l \subset \mathcal{F}_k, 1 \leq l \leq k$ ” into “ $\mathcal{F}_{l-1} \supset \mathcal{F}_k, l \geq k + 1$ ”.
- Page 270 line 2: Delete “for $k \in \{1, 2, \dots, B - 1\}$ ”.
- Page 280 line 12: Change “ $M_n \geq x$ ” to “ $\phi(M_n) \geq x$ ”.
- Page 281 line 1: “Spatial Poisson processes”.
- Page 281 line 9: Change “of X ” to “on X ”.
- Page 282 line 1 and page 285 line -5: Change “A a (measurable) subset” into “a (measurable) subset A ”.
- Page 285 line 2: “Consider a one-to-one mapping”.
- Page 286 line 2: Change “of X ” to “of Y ”.
- Page 292 line -2: Change “ $\lim_{t \searrow 0} tR(t) = 0$ ” into “ $\lim_{t \searrow \infty} tR(t) = 0$ ”.
- Page 293 Exercise 12.1: Change “ $x^{\beta-1}e^{-t^\beta}$ ” into “ $x^{\beta-1}e^{-x^\beta}$ ”.
- Page 312 line 7: Change “through and the set” into “through state”
- Page 312 line 9: $g_k := \mathbb{P}(X_{T_A} = 13 \mid X_0 = k)$.
- Page 312 line -9: Change “through and the set” into “through state”
- Page 312 line -7: Change “ $g(k)$ ” to “ g_k ”.
- Page 314 line -2: analysis.
- Page 315: From node 2 to itself the probability should be 0.4, not 0.3, and from node 2 to node 1 it should be 0.3, not 0.4.
- Page 318 line 9, Problem 5.22-a): It should be “we have $h(0) = 0$ ”.
- Page 319 line 7: Remove a “ $40(70 - 4)$ ” due to repetition.
- Page 320 line 9: greatest.
- Page 320 lines -5 and -4: Change “ $T_0^r = \infty$ ” to “ $T_0^r < \infty$ ” four times.
- Page 324 line -3: Change “ X_{n-1} ” to “ X_{n+1} ”.
- Page 325 line 2: Change “ $\{1, 2, 3\}$ ” to “ $\{1, 2, 3, 4\}$ ”.

- Page 327: Change the notation “ $T_{A,B}$ ” to “ $T_{\{A,B\}}$ ” in 12 places.
- Page 328 line 7: respectively.
- Page 331 line -6: Change “ $\mathbb{P}(N = 2) = \beta(1 - \beta)$ ” to “ $\mathbb{P}(N \geq 2) = 1 - \beta$ ”.
- Page 334 lines 9,12,14: It is better to replace k_1 with i and k_n with j .
- Page 335 line -3: Change “ $h(i)$ ” to “ $h_{n+1}(i)$ ”.
- Page 336 line 8: Change “ $\mathbb{E}[X_n]$ ” to “ $\mathbb{E}[X_n | X_0 = i]$ ”.
- Page 341 line -2: “ $\mathbb{P}(Z = n) \mathbb{E}[U_1] \mathbb{E}[Z]$ ” should be “ $\mathbb{P}(Z = n) = \mathbb{E}[U_1] \mathbb{E}[Z]$ ”.
- Page 342 line 6 of Exercise 9.1: Change “ $\{0, \dots, k\}$ ” to “ $\{1, \dots, k\}$ ”.
- Page 344 line 1: Add “ $= \mathbb{P}(N_t^R = 0)$ ”.
- Page 344 line 11: Change “ $N_t^R + Y$ ” to “ $N_t^R + N_t^W$ ”.
- Page 345, upper half: It is better to change the notation X_λ, X_μ to τ_λ, τ_μ .
- Page 350 line 12: Change “ T_0 ” to “ T_1 ” and “ T_1 ” to “ T_2 ”.
- Page 351 Exercise 9.14-c), the same conclusion follows by adding the extra crossing time T to the answer of Question (b).
- Page 354 Exercise 9.17 part (b): Remove $\alpha \mathbb{E}[T_1^r | X_0 = 1]$ on lines -7 and -6, and delete line -4.
- Page 358 line -10: Change the second last term “ $(\mu + \lambda)\lambda^{k-1}\lambda^{N-k}$ ” to “ $(\mu + \lambda)^k \lambda(\mu + \lambda)^{N-k-1}$ ”.
- Page 358 line -5: Change “negative” to “non-negative”.
- Page 358 line -1: Change “ \leq ” to “ \geq ”.
- Page 359 line -7: Change “ $(M_n)_{n \in \mathbb{N}}$ is a martingale” to “ $(N_n)_{n \in \mathbb{N}}$ is a martingale”.
- Page 363 Reference [Lal]: Change “/lalley/” to “/~lalley/”.
- Pages 164, 165, 169, 195, 292, the reference to Karlin & Taylor (1981) should be changed to:
Karlin, S. and Taylor, H. (1998). An introduction to stochastic modeling. Academic Press, Inc., San Diego, CA, third edition.