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This text is an introduction to the pricing and hedging of financial derivatives, including vanilla and exotic options, by stochastic calculus and partial differential equation methods. The presentation is done both in discrete and continuous-time financial models, with an emphasis on the complementarity between algebraic and probabilistic methods. In particular it covers the pricing of some interest rate derivatives, of American options, of exotic options such as barrier, lookback and Asian options, and stochastic models with compound Poisson jumps. The text is accompanied with a number of figures and simulations, and includes numerous examples based on actual market data. The concepts presented are also illustrated by 381 figures, and 277 exercises and 18 problems with complete solutions. It also includes 30 Python codes and 85 R coding examples for illustrations based on market data.